

## Polycyclic groups of diffeomorphisms on the half-line

Yoichi MORIYAMA

(Received August 25, 1993)

### Introduction

Groups of diffeomorphisms of one-dimensional manifolds are connected with codimension one foliations and present interesting facts. Polycyclic groups of diffeomorphisms of the real line are studied by J. F. Plante [P1], [P3]. Their results are applied to codimension one foliations on manifolds with solvable fundamental groups (see S. Matsumoto [Ma] and [P3]). We are interested in the case where the groups have fixed points. This case reduces to the groups of diffeomorphisms of the half-line. For these groups in case where they are abelian, several facts are already known. We are concerned with both abelian and non-abelian cases in this paper. Partial results for polycyclic groups of diffeomorphisms of the half-line are obtained by Plante [P2], Plante and Thurston [P-T]. Our results describe the classification of such polycyclic groups, that is, polycyclic groups of the diffeomorphisms on the half-line can be essentially classified into two types. The main result is the following.

**THEOREM.** *Let  $\Gamma$  be a polycyclic subgroup of  $\text{Diff}^r[0, \infty)$ ,  $N$  the nilradical of  $\Gamma$  and let  $r = 2, \dots, \infty$ . Assume that  $\text{Fix}(\Gamma) (= \{x \in [0, \infty) \mid f(x) = x \text{ for any } f \in \Gamma\}) = \{0\}$ . Then the following hold.*

(i) *If  $\text{Fix}(N) = \{0\}$ , then  $\Gamma|_{(0, \infty)}$  is  $C^r$  conjugate to a subgroup of the group  $\text{Aff}^+(\mathbf{R})$  of the orientation preserving affine maps of the real line.*

(ii) *If  $\text{Fix}(N) \neq \{0\}$ , then there exists a contraction  $f \in \text{Diff}^r[0, \infty)$  such that  $\Gamma$  is isomorphic to a semi-direct product  $N \rtimes Z_f$  of  $N$  and  $Z_f$  where  $Z_f$  denotes the infinite cyclic group generated by  $f$ .*

For the detailed definitions, see Sections 1, 5 and 6. The proof of the theorem is in Section 6. Examples of the polycyclic groups are given in Section 5.

The author would like to express his hearty thanks to Professor Haruo Suzuki and Professor Toshiyuki Nishimori for helpful conversations and valuable comments.