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On split, separable subalgebras with counitality condition

In memory of Oscar Goldman

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Abstract. A natural algebraic generalization of V.F.R. Jones' theory of subfactors is defined and studied. Noncommutative finite separable extensions of K-algebras are defined from the algebraic notions of relative separability, split extension, and a counit condition. Examples are drawn from group, field and general Galois theory, which is of interest due to the existing comparisons between Jones' subfactor theory and these other algebraic theories. Finite separable extensions possess the main properties of the subfactor theory such as index and iterative aspects that lead to a tower of algebras and braid group representations. We prove that global dimension and other homological properties are the same for overalgebra and subalgebra in a finite separable extension.

Key words: finite separable extension, Galois extension, conditional expectation, index, endomorphism ring, global dimension, braid group.

1. Introduction

M. Pimsner and S. Popa took up a study in [26] of index and algebraic structure in the type II_1 subfactor theory pioneered by Jones [13]. A question that appears implicitly in their article asks what properties are shared by a subalgebra S and an algebra A given the structure of a separable Frobenius extension. Pimsner and Popa had proved that the type II_1 factor von Neumann algebra pairs $N \subseteq M$ under study are finite projective extensions, but they provided formulas indicating something rather stronger: the algebra pairs are separable Frobenius extensions (as developed in [18], [30] and [33]). We show that $N \subseteq M$ is something even stronger than separable Frobenius: M is a split, separable extension of N with counitality condition. In this paper, we define and make an algebraic study of such extensions, which we call finite separable extensions. We prove in Theorem 4.2 that the endomorphism ring of the extension is itself a finite separable extension of the overalgebra, a type of endomorphism ring theorem such as the one in [19] and [21]. On the one hand, the endomorphism ring theorem

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