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Asymptotic behaviors of radial solutions to semilinear wave equations in odd space dimensions

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0. Introduction

This paper is concerned with semilinear wave equations of the form

$$u_{tt}-u_{rr}-\frac{n-1}{r}u_{r}=F(u, u_{t}, u_{r}) \quad \text{in} \quad \boldsymbol{R}^{2}, \tag{0.1}$$

where $u=u(r, t)$ is a real-valued function and $n=2m+3$ with m a non-negative integer. For a large class of the nonlinear term F we will show that "small" solutions of (0.1) exist and are asymptotic to the solutions of the linear wave eqaution

$$u_{tt}-u_{rr}-\frac{n-1}{r}u_{r}=0 \quad \text{in} \quad \boldsymbol{R}^{2}, \tag{0.2}$$

namely, there exist solutions u_{-}, u_{+} of (0.2) and $u(t)-u_{\pm}(t)\rightarrow 0$ as $t\rightarrow\pm\infty$ in the sense of the energy norm.

As is well known, the equation (0.1) is the radially symmetric version of a special case of

$$u_{tt}-\Delta u=F_{0}(u, Du, D_{x}Du) \quad \text{in} \quad \boldsymbol{R}^{n}\times\boldsymbol{R}, \tag{0.3}$$

where $D=(D_{x}, D_{t})$, $D_{x}=(\partial/\partial x_{1}, \ldots, \partial/\partial x_{n})$ and $D_{t}=\partial/\partial t$. The existence of global small solutions of the Cauchy problem for (0.3) has been shown by Christodoulou [3], Li Ta-tsien and Chen Yun-mei [11], and Li Ta-tsien and Yu Xin [12], provided the nonlinear term F_{0} and the initial data prescribed on $t=0$ are "nice". Moreover the asymptotic behaviors as $t\rightarrow\pm\infty$ for solutions of (0.3), which guarantee the existence of the scattering operator, have been researched by Strauss [19], Mochizuki and Motai [13], [14], Pecher [15], Tsutaya [22] and Kubota and Mochizuki [10] in the case where F_{0} is independent of Du, $D_{x}Du$, *i. e.*, $F_{0}=F_{1}(u)$, and by Klainerman [7], Shatah [16] and Klainerman and Ponce [8] in the case where F_{0} does not explicitly depend on u, *i. e.*, $F_{0}=F_{2}(Du, D_{x}Du)$.

In the present paper we study the asymptotic behaviors of radial solutions to (0.3), which guarantee the existence of the scattering opeartor, in