

The Connes spectrum for actions of compact Kac algebras and factoriality of their crossed products

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Abstract. In this paper, we introduce the Connes spectrum for actions of compact Kac algebras on von Neumann algebras. Among other things, it is shown that the crossed product by an action of a compact Kac algebra is a factor if and only if the action is centrally ergodic and has full Connes spectrum.

Key words: von Neumann algebras, Kac algebras, actions, Connes spectrum, crossed products.

Introduction

It is widely acknowledged that the Arveson-Connes theory of the spectrum for actions of locally compact abelian groups on von Neumann algebras was highly successful and was a principal tool for the structure analysis of factors of type *III* (see [C], [CT]). It is no doubt that to look at the (Connes) spectrum is always very essential to have a deep understanding of such actions. Later, this theory was effectively extended by Olesen-Pedersen (see [Ped]) (also by Kishimoto [Ki]) to the case of abelian actions on C^* -algebras in order to investigate the (ideal) structure (i.e., primeness or simplicity) of the crossed product algebras. Definitions in the case of a non-abelian (compact) group action were presented both in the C^* and W^* -situations [EvS], [K], [GLP]. It seems however that the definition employed in [GLP] is a “best” one in the C^* -case in the sense explained in the introduction of [GLP]. At the same time, this spectrum theory was generalized also to the case of group coactions on operator algebras in [K], [N] (see also [Q] for the discrete case). So one would naturally expect that there should be a unified approach to both situations. Our purpose of this article is to extend this generalization program as far as the case of an action of a compact Kac algebra on a von Neumann algebra. As noted in the introduction of [GLP], in a “good” definition of the spectrum, the kind of result one would expect to generalize in the W^* -case is the theorem of Connes and Takesaki in [CT,