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Strongly orthogonal subsets in root systems

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Abstract. We classify maximal strongly orthogonal subsets (= MSOS's) in irreducible root systems under the action of Weyl groups, including non-reduced cases. We show that among irreducible root systems, B_n (n = *even*), C_n ($n \geq 2$), F_4 and BC_n ($n \geq 1$) admit several inequivalent MSOS's. As an application of this result, we give a classification of MSOS's associated with Riemannian symmetric pairs.

Key words: root system, strongly orthogonal subset, Riemannian symmetric space.

Introduction

In our paper [1], in order to solve a geometric problem concerning the existence of local isometric imbeddings of compact irreducible Riemannian symmetric spaces G/K, we constructed subsets Γ of root systems Δ having the following properties:

(C.1) $\theta\alpha = -\alpha$ for all $\alpha \in \Gamma$, where θ means the involution of Δ induced from the symmetry of G/K.
(C.2) If $\alpha, \beta \in \Gamma$, $\alpha \neq \beta$, then $\alpha \pm \beta \notin \Delta \cup \{0\}$.
(C.3) It holds $\#\Gamma = s(G/K)$, where $s(G/K) = \mathrm{rank}(G/K) - \mathrm{rank}(G) + \mathrm{rank}(K)$.

Using these subsets Γ satisfying the above conditions, we determined the maximum of the rank of the curvature transformation of G/K, and gave some estimates on the dimension of the Euclidean space into which G/K can be locally isometrically immersed.

In [2] we introduced a new geometric quantity $p(G/K)$ naturally associated with G/K, by which we improved the estimates given in [1]. For example, by calculating the value $p(G/K)$ for $G/K = Sp(n)$, we proved that the canonical imbedding of the symplectic group $Sp(n)$ into $\boldsymbol{R}^{4n^2}$ gives the least dimensional isometric imbedding even in the local standpoint (see [3]). It is desirable to determine the value $p(G/K)$ for all Riemannian symmetric spaces, though it is a considerably difficult algebraic problem.

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