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A note on the large time asymptotics for a system of Klein-Gordon equations

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Abstract. We study the asymptotic behavior of the solution in the large time for the system

$$\begin{cases} (\square + m_1^2)u_1 = 0 \\ (\square + m_2^2)u_2 = 0 \\ (\square + m_3^2)u_3 = u_1u_2 \end{cases}$$

in two space dimensions. We prove the existence of a solution whose amplitude is modulated by the long range effect when $m_3 = \lambda_1 m_1 + \lambda_2 m_2$ for some λ_1, $\lambda_2 \in \{\pm 1\}$.

Key words: Klein-Gordon equation; asymptotic behavior.

1. Introduction

We are concerned with the large time behavior of the solution to the Cauchy problem

$$\begin{cases} (\square + m_i^2)u_i = F_i(u_1, \dots, u_N), & t > 0, x \in \mathbf{R}^n, \\ (u, \partial_t u)|_{t=0} = (\varepsilon f_i, \varepsilon g_i), & i = 1, \dots, N. \end{cases} \tag{1.1}$$

Here $\square = \partial_t^2 - \Delta_x$, $m_i > 0$ is a constant, f_i and g_i are real valued functions which belong to the Schwartz class $\mathcal{S}(\mathbf{R}^n)$, and $\varepsilon > 0$ is a small parameter. We also assume that for each $i \in \{1, \dots, N\}$, F_i is a smooth function of $u = (u_1, \dots, u_N)$ in its argument and vanishes to p-th order at the origin, i.e.

$$\frac{\partial^\alpha F_i}{\partial u^\alpha}(0) = 0 \quad \text{for} \quad |\alpha| \leq p - 1.$$

Since we are interested in the critical nonlinear case, we restrict ourselves to the case $p = 1 + 2/n$ and $n \leq 2$, that is, $(n, p) = (1, 3)$ or $(n, p) = (2, 2)$.

In the previous work [8], it is shown that the Cauchy problem (1.1) admits a unique global smooth solution which tends to a free solution as $t \to \infty$ under the following condition, which we call *the non-resonance*

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