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Gluck Twist on a Certain Family of 2-Knots

Daniel Nash & András I. Stipsicz

1. Introduction

The paper of Freedman, Gompf, Morrison, and Walker [5] about the potential application of Khovanov homology in solving the 4-dimensional smooth Poincaré conjecture (SPC4) revitalized this important subfield of topology. A sequence of papers appeared, some settling 30-year-old problems [1; 7], some introducing new potential exotic 4-spheres [14], and still others showing that the newly introduced examples are, in fact, standard [2; 15].

One underlying construction for producing examples of potential exotic 4-spheres is the Gluck twist along an embedded S^2 (a *2-knot*) in the standard 4-sphere $\mathbb{S}^4$. In this construction we remove the tubular neighborhood of the 2-knot and glue it back with a specific diffeomorphism. (For a more detailed discussion, see Section 3.) In turn, any 2-knot in $\mathbb{S}^4$ admits a *normal form* and hence can be described by an ordinary knot in S^3 together with two sets of ribbon bands (determining the "southern" and "northern" hemispheres of the 2-knot). Applying standard ideas of Kirby calculus (see e.g. [8]), we can explicitly draw the complement of a 2-knot and, from there, the result of the Gluck twist. From such a presentation we derive the following result.

Theorem 1.1. *Consider the knot $K(p,q)$ depicted by Figure 1, and use the bands b_1 and b_2 to construct the southern and northern hemispheres of a 2-knot $K^2_{pq} \subset \mathbb{S}^4$. Then a Gluck twist along the 2-knot K^2_{pq} provides the 4-sphere with its standard smooth structure.*

Remark 1.2. For the first appearance of the 1-knots $K(p,q)$ see [11; 12]. For certain choices of p and q, the 1-knot $K(p,q)$ can be identified more familiarly; for instance, $K(0,0)$ is isotopic to $F\#F = F\#m(F)$, where F is the figure-eight knot (isotopic to its mirror image $m(F)$), $K(1,-1)$ is the 8_9 knot, and $K(1,1)$ is 10_{155} in the standard knot tables. Notice that in [3] the knot 8_9 defines the 2-knot along which the Gluck twist is performed, although the bands used in [3] are potentially different from the b_1 and b_2 used here in Theorem 1.1 (cf. [3, Fig. 16]).

Before proving the theorem, in Sections 2 and 3 we briefly invoke basic facts about 2-knots, the Gluck twist, and the derivation of a Kirby diagram for the result of the Gluck twist along a 2-knot given by a ribbon 1-knot and two sets of

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