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A Family of Knots Yielding Graph Manifolds by Dehn Surgery

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Dedicated to Professor Yukio Matsumoto for his 60th birthday

1. Main Theorem

Let $P_{(l,r)}$ be an embedded once-punctured torus, $k_{(l,a;r,b)}$ a knot in $P_{(l,r)}$ in S^3 defined as in Figure 1, and

$$p_{(l,a;r,b)} := la^2 + ab + rb^2,$$

where (a,b) is a coprime pair of integers a,b with $1 < a < b$ and where l and r are integers. We will study the knots $k_{(l,a;r,b)}$ themselves later. Our main theorem concerns Dehn surgery along $k_{(l,a;r,b)}$.

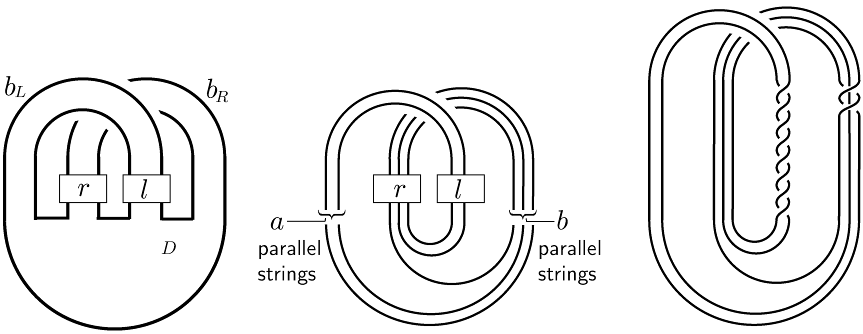

Figure 1 $k_{(l,a;r,b)}$ in $P_{(l,r)}$ (here, $k_{(4,2;1,3)}$)

Theorem 1.1. *For each $(l,a;r,b)$ as described previously, the resulting manifold $(k_{(l,a;r,b)}; p_{(l,a;r,b)})$ of $p_{(l,a;r,b)}$-surgery along the knot $k_{(l,a;r,b)}$ is "at most" a graph manifold obtained by splicing two Seifert manifolds over S^2 (possibly reduced to a Seifert manifold over S^2, a lens space, or a connected sum of two lens spaces in some cases).*

In fact, $(k_{(l,a;r,b)}; p_{(l,a;r,b)})$ bounds a plumbing manifold [O, p. 22] corresponding to the weighted graph in Figure 2; that is, $(k_{(l,a;r,b)}; p_{(l,a;r,b)})$ is described by the framed link in the figure. We will give an algorithm to decide the integers n_L, n_R

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