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On the Boundary Accumulation Points for the Holomorphic Automorphism Groups

Jisoo Byun

1. Introduction

For a domain Ω in $\mathbb{C}^n$, we denote by $\mathrm{Aut}(\Omega)$ the group of holomorphic automorphisms of Ω. It is obvious that $\mathrm{Aut}(\Omega)$ is a topological group with respect to the law of composition and the compact-open topology. In particular, it is a theorem of H. Cartan that $\mathrm{Aut}(\Omega)$ is in fact a Lie group if Ω is bounded.

In light of the outstanding question "Which domains possess noncompact automorphism group?" there is much interest focused upon the existence and nonexistence of orbits of the automorphism group action accumulating at a given boundary point. The well-known Greene–Krantz conjecture belongs to such a line of research. In this paper, we discuss the *finite*-type boundary points that repel automorphism orbits.

Denote by $\tau_\Sigma(q)$ the D'Angelo type (see [10]) at q of the real hypersurface Σ in $\mathbb{C}^n$. Now we present our main theorem.

Theorem 1.1. *Let Ω be a domain in $\mathbb{C}^2$. Assume that there exists a point $p \in \partial\Omega$ admitting an open neighborhood U in $\mathbb{C}^2$ satisfying the conditions*

(1) *the boundary $\partial\Omega$ is C^∞-smooth pseudoconvex in U, and*
(2) *$\tau_{\partial\Omega}(q) < \tau_{\partial\Omega}(p) < \infty$ for every $q \in U \cap \partial\Omega \setminus \{p\}$.*

Then there are no automorphism orbits in Ω accumulating at p.

In particular, this implies the following theorem of Byun.

Theorem 1.2 [8]. *In the Kohn–Nirenberg domain defined by the inequality*

$$\operatorname{Re} w + |zw|^2 + |z|^8 + \frac{15}{7}|z|^2 \operatorname{Re} z^6 < 0,$$

there does not exist any automorphism group orbit accumulating at the origin.

Although several experts commented that the nonconvexifiability of the boundary at the origin should be the reason for the conclusion of Byun's theorem, it is now apparent by our main theorem that the essential reason in fact lies elsewhere: any isolated maximum finite-type boundary point repels automorphism orbits.

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