Sampling Sequences for $A^{-\infty}$

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I. Introduction

For every $n > 0$, we define A^{-n} to be the Banach space of all functions f analytic in the unit disc U such that

$$\|f\|_{A^{-n}} \equiv \sup_{z\in U} |f(z)|(1-|z|^2)^n < \infty.$$

If $f \in A^{-n}$ and if $\Gamma \subset U$ is any subset then we can define

$$\|f|_\Gamma\|_{A^{-n}} = \sup_{z\in\Gamma} |f(z)|(1-|z|^2)^n.$$

Thus we always have

$$\|f|_\Gamma\|_{A^{-n}} \le \|f\|_{A^{-n}}.$$

Γ is called an A^{-n} *sampling set* if there exists a constant L such that, for every $f \in A^{-n}$,

$$\|f\|_{A^{-n}} \le L\|f|_\Gamma\|_{A^{-n}}.$$

The smallest such L, designated $L(\Gamma, n)$ is called the *sampling constant* of Γ. In an important paper, Seip [4] gave a complete characterization of A^{-n} sampling sets in terms of a certain density that he defined.

The space $A^{-\infty}$ is defined by

$$A^{-\infty} = \bigcup_{n>0} A^{-n};$$

that is, it is the algebra of functions analytic in U satisfying

$$|f(z)| \le \frac{M}{(1-|z|)^n} \quad \text{for some constants } M \text{ and } n.$$

Equipped with the inductive limit topology, $A^{-\infty}$ becomes a topological algebra. The zero sets and closed ideals of $A^{-\infty}$ were completely characterized in [2] and [3].

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