A Characterization of Hilbert Spaces in Terms of Multipliers between Spaces of Vector-Valued Analytic Functions

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0. Introduction

Given a complex Banach space $(X, \|\cdot\|)$, we shall denote by $H^1(X)$ the space of X-valued Bochner integrable functions on the circle $\mathbb{T} = \{|z| = 1\}$ whose negative Fourier coefficients vanish; that is,

$$H^1(X) = \{f \in L^1(\mathbb{T}, X) \colon \hat{f}(n) = 0 \text{ for } n < 0\}.$$

We write

$$\|f\|_{1,X} = \int_0^{2\pi} \|f(e^{it})\| \frac{dt}{2\pi}$$

for the norm in $H^1(X)$.

We shall also denote by $\mathrm{BMOA}(X)$ the space of vector-valued BMO functions on the circle with analytic extension to the unit disk D; that is, $f \in L^1(\mathbb{T}, X)$ with $\hat{f}(n) = 0$ for $n < 0$ such that

$$\|f\|_{*,X} = \sup_I \left(\frac{1}{|I|} \int_I \|f(e^{it}) - f_I\|^2 \frac{dt}{2\pi} \right)^{1/2} < \infty,$$

where the supremum is taken over all intervals $i \in \mathbb{T}$, $|I|$ stands for the normalized Lebesgue measure of I, and

$$f_I = \frac{1}{|I|} \int_I f(e^{it}) \frac{dt}{2\pi}.$$

The norm in the space is given by

$$\|f\|_{\mathrm{BMO}(X)} = \left\| \int_0^{2\pi} f(e^{it}) \frac{dt}{2\pi} \right\| + \|f\|_{*,X}.$$

Finally, we shall use $\mathrm{Bloch}(X)$ to denote the space of X-valued analytic functions on D, say $f(z) = \sum_{n=0}^{\infty} x_n z^n$, such that $\sup_{|z|<1} (1-|z|)\|f'(z)\| < \infty$. To avoid constant functions have zero norm we consider

$$\|f\|_{\mathrm{Bloch}(X)} = \|f(0)\| + \sup_{|z|<1} (1-|z|)\|f'(z)\|.$$

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