Excision of Equivariant Cyclic Cohomology of Topological Algebras

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1. Introduction

One of the fundamental theorems in cyclic (co-)homology is the excision theorem, which was developed by Wodzicki [19; 21] and generalized to the bivariant case by Kassel [11]. In [11; 18; 20] the excision theorem was used to construct the Chern character from KK-theory to bivariant cyclic theory, and to obtain the vanishing of cyclic (co-)homology of stable $\mathbf{C}^*$-algebras. In this paper we define equivariant cyclic (co-)homology of topological algebras with compact group actions and study its excision property. Section 2 is devoted to the basic definitions of equivariant Hochschild and cyclic (co-)homologies, which are motivated by twisted cyclic (co-)homology [8]. This section can be considered as an improvement of Brylinski's equivariant Hochschild homology [3] and a supplement of Klimek–Kondracki–Lesniewski's equivariant entire cyclic cohomology for finite group actions [12]. Our definitions are slightly different from those in [12]. In Sections 3 and 4 we prove our main results, the excision theorems of equivariant Hochschild and cyclic (co-)homologies, by introducing equivariant H-unitality, which is inspired by Wodzicki's theorem [21]. The main point is to deal with the twisting of group actions. As a corollary of the excision theorem, we obtain the six-term exact sequence of equivariant periodic cyclic cohomology. In Section 5 we show the existence of Mayer–Vietoris sequences of equivariant cyclic (co-)homology which were used in the ordinary case to prove that the periodic cyclic homology $PHC_{\mathrm{ev}}(C^\infty(G,M))$ of the algebra $C^\infty(G,M)$ for a compact smooth manifold M is isomorphic to K-theory $K_G^0(M)\otimes_{\mathbf{Z}}\mathbf{C}$ [2; 3]. Finally, we discuss the equivariant H-unitality in Section 6, which is much more difficult than that in [21]. We do not know in general for what algebras we will have the excision property of equivariant cyclic cohomology, although the equivariant H-unitality is quite understandable. The Chern character in equivariant cyclic (co-)homology will be constructed in [9]. The motivation of the present paper is the possible applications to the equivariant Novikov conjecture [5; 6; 10; 15] by means of equivariant cyclic cohomology, which we hope to study subsequently.

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