Intersection Homology and Free Group Actions on Witt Spaces

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1. Introduction

The study of free G actions of a finite group on manifolds has been of great interest to topologists for many years. The space form problem is just one of many problems involving free G actions on manifolds. Poincaré duality has played an important role in studying free G actions on manifolds.

It is natural to try to study free G actions on spaces with singular sets. Witt spaces are a class of PL spaces with singularities that satisfy a Poincaré duality theorem using the intersection homology of Goresky and MacPherson ([7], [8], [24]). Irreducible complex projective varieties are an important class of Witt spaces. Thus Witt spaces are a natural candidate to choose to study free G actions on spaces with singularities. We obtain several intersection homological restrictions on Witt spaces admitting a free G action. All group actions in this paper are assumed to be groups acting through PL homeomorphisms.

The first result is a restriction on the Euler characteristic. If a compact Witt space X^n of dimension n has a free G action that acts trivially on the intersection homology of X, then the Euler characteristic $\sum_{i=0}^{n}(-1)^i \dim(IH_i^{\bar{m}}(X;\mathbf{Q}))$ is zero.

Another restriction is given on the semicharacteristic. Suppose X^{4n+1} is a compact Witt space of dimension $4n+1$, G acts freely on X, and G acts trivially on the intersection homology of X; then either the semicharacteristic $\sum_{i=0}^{2n}(-1)^i \dim(IH_i^{\bar{m}}(X;\mathbf{Q}))$ is even or G is the direct product of a cyclic 2-group and an odd order group.

Another result is a restriction on the higher signature of a compact Witt space X. Let $L(X)$ denote the L class of X as defined by Goresky and MacPherson, and let $f: X \to B\pi$ classify the universal cover of X. Then $f_*(L(X)) \in H_*(B\pi;\mathbf{Q})$ is the higher signature of X. Suppose X has a free G action such that G acts trivially on the fundamental group and trivially on the intersection homology of X with any local coefficient system. Consider a representation $\rho: \pi \to \mathrm{Sp}(2l,\mathbf{R})$ of the fundamental group of X into the real

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