# Multipliers and Cyclic Vectors in the Bloch Space

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## I. Introduction

In this paper we study the cyclic vectors in $\mathcal{B}$, the Bloch space with the weak* topology, and in $\mathcal{B}_0$, the "little" Bloch space with the norm topology. A result is obtained which implies that every outer function in $\mathcal{B}(\mathcal{B}_0)$ is cyclic. We also obtain a simple characterization of multipliers in $\mathcal{B}$ and $\mathcal{B}_0$.

The Bloch space $\mathcal{B}$ in the open unit disc $D$ in the complex plane is the space of all those analytic functions $f$ such that $(1-|z|^2)f'(z)$ is bounded in $D$. We norm $\mathcal{B}$ as follows:

$$\|f\|=|f(0)|+\sup\{(1-|z|^2)|f'(z)|: z\in D\}. \tag{1}$$

With this norm $\mathcal{B}$ is a Banach space and $\mathcal{B}_0$ a closed subspace. Here $\mathcal{B}_0$, sometimes called the "little" Bloch space, denotes the set of those $f$ in $\mathcal{B}$ for which $(1-|z|^2)f'(z)\to 0$ as $|z|\uparrow 1$. For information about $\mathcal{B}$ and $\mathcal{B}_0$, see [1] and [2].

The space $\mathcal{B}$ with the norm (1) is isometric to the second dual $\mathcal{B}_0^{**}$ (see [9]). Furthermore, the polynomials are norm dense in $\mathcal{B}_0$ and in $\mathcal{B}_0^*$, and are weak* dense in $\mathcal{B}$. Note that $\mathcal{B}$ is not norm separable.

We have a growth estimate for Bloch functions (see, e.g., [3, Eq. (4)]):

$$|f(z)|\le\left\{1+\log\frac{1}{1-|z|}\right\}\|f\|. \tag{2}$$

Thus $\mathcal{B}$ is contained in $L_a^p$ (the analytic $L^p$ functions in $D$) for $p<\infty$. For the Hardy spaces we have $H^\infty\subset\mathcal{B}$, but $H^p$ is not contained in $\mathcal{B}$ for any $p<\infty$; also, $\mathcal{B}$ is not contained in the Nevanlinna class.

The second section of this paper deals with multipliers in $\mathcal{B}$. A complex-valued function $\phi$ in $D$ is called a *multiplier on* $\mathcal{B}$ if $\phi\mathcal{B}\subset\mathcal{B}$. By $M_\phi$ we denote the operator of multiplication by $\phi: M_\phi f=\phi f$ $(f\in\mathcal{B})$. The set of all multipliers will be denoted by $M(\mathcal{B})$. An application of the closed graph theorem shows that $M_\phi$ is a bounded linear transformation on $\mathcal{B}$. Hence it has a finite norm $\|M_\phi\|$.

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