THE BOUNDARY BEHAVIOR OF BLOCH FUNCTIONS AND UNIVALENT FUNCTIONS

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1. Introduction. The function $f(z)=\sum_{n=0}^{\infty}a_nz^n$, analytic for $z\in D=\{z:|z|<1\}$, is called a Bloch function if the norm

$$\|f\|=\sup\{(1-|z|^2)|f'(z)|:z\in D\}+|f(0)|$$

is finite. The set of such functions forms a Banach space B and the subspace of B consisting of those $f\in B$ for which $(1-|z|^2)|f'(z)|\to 0$ as $|z|\to 1-$ is denoted by B_0. The Zygmund class Λ^* consists of those (complex-valued) continuous functions $F(t)$ of period 2π for which

$$F(t+h)+F(t-h)-2F(t)=O(|h|)$$

uniformly in t. If the above second difference is $o(|h|)$ as $|h|\to 0$ then we say that $f\in\lambda^*$.

The spaces Λ^* and λ^* are also Banach spaces with the obvious norm. Moreover,

$$f(z)=\sum_{n=0}^{\infty}a_nz^n\in B \Leftrightarrow F(t)=\sum_{n=1}^{\infty}\frac{a_n}{n}e^{int}\in\Lambda^* \tag{1}$$

and similarly $f\in B_0$ if and only if $F\in\lambda^*$. Thus the space B is isomorphic under the operation of integration to that subspace of Λ^* consisting of functions whose negative Fourier coefficients vanish. In [2] we considered the spaces of real-valued functions in Λ^* and λ^*, but in the present paper we study the complex case. This eventually resolves itself into a consideration of the radial or boundary behavior of Bloch functions and univalent functions.

As in [1], an important class of examples is given by lacunary series

$$f(z)=\sum_{k=0}^{\infty}a_kz^{n_k},$$

where $n_{k+1}/n_k\geq q>1$ for all k, and

$$\|(a_k)\|_\infty=\sup\{|a_k|:k\geq 0\}$$

is finite. Such functions belong to B and also to B_0 if $a_k\to 0$ as $k\to\infty$. If, moreover,

$$\sum_{k=0}^{\infty}|a_k|^2=\infty, \tag{2}$$

then such functions $f(z)$ have finite radial limits $\lim_{r\to 1-}f(re^{i\theta})$ only for a set E of values of θ of measure zero. Also the corresponding functions $F(t)$ have finite derivatives only in such a set E.

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