EXCEPTIONAL SETS FOR HOLOMORPHIC SOBOLEV FUNCTIONS

Patrick Ahern

We let B^n denote the unit ball in $\mathbf{C}^n$ and let S denote its boundary. For the most part we will follow the notation and terminology of Rudin [11]. For f holomorphic in B^n the radial derivative of f is defined to be

$$Rf(z)=\sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z)=\sum_{k=0}^{\infty} k f_k(z),$$

if $f=\sum f_k$ is the homogeneous polynomial development of f. For $\beta>0$ one is led to the definition $R^\beta f(z)=\sum_{k=0}^{\infty}(k+1)^\beta f_k(z)$ (see [6]). For $p,\beta>0$ we define

$$H_\beta^p(B^n)=\{f\in H(B^n): R^\beta f\in H^p(B^n)\},$$

where $H^p(B^n)$ is the usual Hardy space [11]. $H_\beta^p(B^n)$ may be considered as a holomorphic version of a Sobolev space [6]. For $\zeta\in S$ and $\delta>0$ there is the Koranyi ball $B(\zeta,\delta)=\{\eta\in S: |1-\langle\zeta,\eta\rangle|<\delta\}$. There are also the admissible approach regions for $\zeta\in S$, $\alpha>1$,

$$D_\alpha(\zeta)=\{z\in B^n: |1-\langle z,\zeta\rangle|<(\alpha/2)(1-|z|^2)\}.$$

For each function $f: B^n\to\mathbf{C}$ we have the admissible maximal function

$$M_\alpha f(\zeta)=\sup_{z\in D_\alpha(\zeta)}|f(z)|.$$

The main result of this paper is the following "trace" theorem for Sobolev functions, which will be proved in Section 1.

THEOREM 1.1. *Suppose that $0<p\le 1$ and $f\in H_\beta^p(B^n)$, where $m=n-\beta p>0$, and that ν is a positive Borel measure on S that satisfies*

$$(*)\qquad \nu(B(\zeta,\delta))\le C\delta^m \quad \textit{for some constant } C.$$

Then for each $\alpha>1$ there is a constant $C=C(\alpha)$ such that

$$\int M_\alpha f(\zeta)^p\, d\nu(\zeta)\le C\|R^\beta f\|_p^p.$$

(Here $\|g\|_p$ denotes the norm of g in $H^p(B^n)$.)

Of course, the strong inequality of the theorem gives rise to a corresponding weak estimate which in turn yields the following.

COROLLARY 1. *Every $f\in H_\beta^p(B^n)$ has an admissible limit almost everywhere, $d\nu$.*

Received February 2, 1987. Revision received May 27, 1987.
The author was partially supported by a grant from NSF.
Michigan Math. J. 35 (1988).