A RESIDUE FORMULA FOR HOLOMORPHIC FOLIATIONS

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INTRODUCTION

In this note we give an explicit formula for the residues of certain of the secondary characteristic classes for holomorphic foliations with trivial normal bundle. These residues exist when the foliation is preserved by a transversely holomorphic vector field. We assume that the vector field has a particularly nice singular set, which allows us to compute some of the residues. The computation of these residues gives a geometric interpretation of the secondary classes, as the form the residues take depends strongly on the local geometry of the vector field and foliation in a neighborhood of the singular set.

Throughout the paper we assume that the reader is very familiar with the construction of characteristic classes using connections and invariant polynomials on Lie groups as given in [9]. In particular note that we observe the Chern convention that if an invariant polynomial does not have enough arguments the last one is repeated until it does; i.e., if ϕ is an invariant polynomial on $gl_q C$ of degree k then

$$\phi(A,B) = \phi(A, \underbrace{B, B, \ldots, B}_{k-1})$$

where $A,B \in gl_q C$.

1. THE RESIDUE THEOREM

Let M be a complex analytic manifold of complex dimension n and

$$T_C M = TM \oplus \bar{T}M$$

the standard splitting of the complexified tangent bundle of M. If ξ is a bundle over M, we denote the space of smooth sections of ξ by $C^\infty(\xi)$. The space of smooth complex valued forms on M is denoted $A(M)$ and the space of smooth complex functions is denoted $C^\infty(M)$.

Let τ be a complex analytic foliation of complex codimension q on M. At each point $z \in M$ there is a coordinate chart $(U, z_1, \ldots, z_n)$ so that $\tau|_U$ is spanned by $\partial/\partial z_{q+1}, \ldots, \partial/\partial z_n$. We call such a chart a flat chart for τ at z. The normal

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