THE ESSENTIAL SPECTRUM OF A HANKEL OPERATOR WITH PIECEWISE CONTINUOUS SYMBOL

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A Hankel operator S on a complex Hilbert space with complete orthonormal basis $\{e_n; n = 0, 1, 2, \dots\}$ is one whose representing matrix has the form

$$S_{ij} = c_{i+j}, \qquad i, j = 0, 1, 2, \dots .$$

A classical theorem of Nehari [6] shows that a sequence $(c_n)_{n=0}^{\infty}$ defines a bounded Hankel operator if and only if it is the sequence of positive Fourier coefficients of an essentially bounded measurable function ϕ on the unit circle. Hartman subsequently showed that S is compact if and only if ϕ can be chosen to be continuous (see [4] or [1]).

In this note we determine the essential spectrum of S when ϕ is a function possessing left and right limits at every point on the circle.

Notation. Let L^2 be the Hilbert space of square integrable functions on the unit circle T with the usual orthonormal basis $\{z^n; n = 0, \pm 1, \pm 2, \dots\}$. The unitary operator J on L^2 is defined by $Jz^n = z^{-n}$ and we shall let P denote the orthogonal projection of L^2 onto the Hardy subspace H^2 spanned by $\{z^n; n = 0, 1, 2, \dots\}$.

For an essentially bounded measurable function ϕ in L^∞, the Toeplitz operator T_ϕ, on H^2, is defined by $T_\phi = PM_\phi \mid H^2$ where M_ϕ is the usual multiplication operator on L^2. We call ϕ the symbol of the Toeplitz operator T_ϕ. The Hankel operator on H^2, with symbol ϕ in L^∞, is defined by $S_\phi = PJM_\phi \mid H^2$.

Let PC denote the collection of functions on T which possess left and right limits at each point. For ϕ in PC and α in T we shall write

$$\phi_\alpha = \frac{1}{2} \lim_{t \to 0+} \{\phi(\alpha e^{it}) - \phi(\alpha e^{-it})\}$$

and call ϕ_α the jump of ϕ at α.

Let T′ denote the non-real points of T and, for $\gamma, \nu \in \mathbb{C}$, let $[\gamma, \nu]$ denote the line segment joining γ and ν. We shall prove the following:

THEOREM 1. Let ϕ be a function in PC. Then

$$\sigma_e(S_\phi) = [0, i\phi_1] \cup [0, i\phi_{-1}] \cup \bigcup_{\alpha \in T'} [-(-\phi_\alpha \phi_{\bar\alpha})^{1/2}, +(-\phi_\alpha \phi_{\bar\alpha})^{1/2}].$$

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