ON THE BREAKDOWN PHENOMENA OF SOLUTIONS OF QUASILINEAR WAVE EQUATIONS

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0. INTRODUCTION

We consider the quasilinear wave equation $y_{tt} - Q^2(y_x)y_{xx} = 0$ subject to the initial and boundary conditions: $y_x(x, 0) = g(x)$, $y_t(x, 0) = f(x)$, $0 \leq x \leq L$; $y(0, t) = y(L, t) = 0$, $t \geq 0$. We can transform this system to the initial-value problem of a hyperbolic conservation law if $f(x)$ and $g(x)$ satisfy some compatibility conditions.

We consider two cases:

Case 1. $g(x) \equiv 0$ and $f(x)$ satisfies some convexity conditions;

Case 2. $f(x) \equiv 0$ and $g(x)$ satisfies some convexity conditions.

We prove that a necessary condition for the existence of a C^2 global solution is that the solution be periodic in t in some sense, which is the classical one for the linear problem. We present a necessary and sufficient set of conditions for the solution to break down in the sense that some second-order derivatives of the solution become unbounded at a finite time. By this set we mean that if the solution breaks down, then one condition in this set holds. Conversely, if one condition in this set holds, then the solution eventually breaks down. We derive some conditions on Q', which are weaker than those considered in [5], [6], and [4], which are sufficient for the solution to break down.

F. John [2] has obtained the breakdown result for the general, genuinely nonlinear conservation laws with n-characteristics and with initial functions which are sufficiently small. For our special conservation law, the genuine nonlinearity condition is $Q' \neq 0$. We derive the breakdown results under some conditions on Q' weaker than $Q' \neq 0$.

1. DEFINITIONS AND NOTATION

We define $u(x, t) = y_x(x, t)$ and $v(x, t) = y_t(x, t)$. The problem is equivalent to the system

$$(1) \qquad \begin{aligned} u_t - v_x &= 0, \\ v_t - Q^2(u)u_x &= 0; \end{aligned}$$

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