FREE INVOLUTIONS ON 6-MANIFOLDS

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INTRODUCTION

In this paper, we give the diffeomorphism classification of smooth, closed, orientable manifolds M of dimension six with $\pi_1 M = Z_2$ and $\pi_2 M = 0$. This is equivalent to the classification of free differentiable orientation-preserving involutions on a connected sum of finitely many copies of $S^3 \times S^3$. In this case, it is therefore possible to carry out the program proposed in [5] for the study of involutions on (n - 1)-connected 2n-manifolds ($n \geq 3$).

The paper is organized as follows. Section 1 contains an explanation of the notation and an exposition of the results needed from [1] and [5]. In Section 2, we state the classification results, Theorems 2 and 3, and give an example. The remaining sections contain the proofs.

1. BILINEAR FORMS

Let K be a finite orientable Poincaré complex of dimension six [8] with $\pi_1 K = Z_2$ and $\pi_2 K = 0$. The generator of $\pi_1 K$ will be denoted by T. Then the integral homology and cohomology groups of the universal covering space $\tilde{K}$ are modules over the integral group ring Λ of Z_2 via the action of T. In particular, $H_3(\tilde{K}) \cong r\Lambda \oplus Z_+ \oplus Z_+$ for some integer r, where Z_+ is the group of integers with trivial action of Z_2. This can easily be shown, if it is recalled that since $H_3(\tilde{K})$ is a free abelian group it has the form $r_0 Z_+ \oplus r_1 Z_- \oplus r_2 \Lambda$ as a Λ-module. From the spectral sequence of the covering $\tilde{K} \to K$, we deduce the values $r_0 = 2$ and $r_1 = 0$.

Let us write $H = H_3(\tilde{K})$ and consider the effect of the involution on the intersection pairing $\lambda: H \times H \to Z$. This is a unimodular, skew-symmetric bilinear form with the further properties

(1) $\lambda(Tx, Ty) = \lambda(x, y)$ for all x, y in H, and

(2) $\lambda(x, x) = \lambda(x, Tx) = 0$ for all x in H.

Associated with λ, there is the Browder-Livesay self-intersection map $\phi: H \otimes Z_2 \to Z_2$ (see [1] and Sections 5 and 6 below). This is related to λ by the equation

$$\phi(x + y) - \phi(x) - \phi(y) = \lambda(x, Ty) \pmod 2,$$

valid for all x, y in H. Although ϕ is actually defined on $H \otimes Z_2$, it will cause no confusion to write $\phi(x)$ for x in H, instead of $\phi(x \otimes 1)$. The geometry of K therefore gives the algebraic data (λ, ϕ, H). Any such triple, satisfying the relations listed above, will be called a Z_2-form.

Received March 13, 1975.
This research was partially supported by NSF Grant GP-38875X.

Michigan Math. J. 22 (1975).