A 3-MANIFOLD ADMITTING A UNIQUE PERIODIC PL MAP

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1. INTRODUCTION

In this paper, we show that a family $\{M(n)\}$ of closed, aspherical 3-manifolds has the property that each $M(n)$ admits a unique PL involution. These 3-manifolds are of special interest, since P. E. Conner and F. Raymond [3] have shown that very few finite groups can act effectively on them. In particular, Z_2 is the only group that can act effectively on $M(1)$. Thus we obtain the following result.

THEOREM 1. *The closed, aspherical* 3-*manifold* M(1) *admits exactly one periodic* PL *map* (*up to conjugation*).

Let T^2 denote the 2-dimensional torus, that is

$$\{(z_1, z_2) \in \mathbb{C} \times \mathbb{C}: |z_1| = |z_2| = 1\}.$$

If n is a positive odd integer, let $\Phi(n)$ denote the homeomorphism $T^2 \to T^2$ defined by

$$\Phi(n)(z_1, z_2) = (z_1^{n-1} z_2, z_1^n z_2).$$

Let R^1 denote the real line, and let $M(n) = (T^2 \times R^1)/\Phi(n)$ be the torus bundle over the circle obtained from $T^2 \times R^1$ by identification of (z_1, z_2, t) with $(\Phi(n)(z_1, z_2), t+1)$.

Denote the points of $M(n)$ by $[z_1, z_2, t]$. Each $M(n)$ admits a standard involution h_0 defined by

$$h_0([z_1, z_2, t]) = [g(z_1, z_2), t],$$

where $g(z_1, z_2) = (\bar{z}_1, \bar{z}_2)$.

Let h be a PL involution of $M(n)$. We obtain the uniqueness of involutions on $M(n)$ by actually constructing an equivalence between h and h_0. Our first step is to obtain an invariant torus fiber T that meets the fixed-point set $\mathrm{Fix}(h)$ of h in exactly four points. Then we split $M(n)$ along T to obtain $T \times [0, 1]$. The involution h defines a product involution $g \times 1$ on $T \times [0, 1]$. If we let ψ denote the homeomorphism repairing the cut made along T, we may view $M(n)$ as $T \times R^1/\psi$. The homeomorphism $\Phi(n)$ is isotopic to a conjugate of ψ, say $\alpha\psi\alpha^{-1}$, where $g\alpha = \alpha g$. In Section 2 we show that this isotopy can be realized by one that commutes with g at each level. We use this equivariant isotopy to define an equivalence between $\alpha h \alpha^{-1}$ and h_0.

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