KÄHLER MANIFOLDS OF CONSTANT NULLITY

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1. INTRODUCTION

The purpose of this paper is to derive curvature conditions that guarantee the existence of a product structure for Kähler manifolds of constant nullity and for Kähler-immersed manifolds of constant relative nullity.

In the intrinsic-manifold case, we shall obtain the following results.

THEOREM A. *Let* M^n *be a complete, connected, and simply connected* C^∞ *Kähler manifold of constant positive nullity* μ, *and suppose that one of the following conditions is satisfied.*

(A1) $\mu = 2$.

(A2) *The restriction of the complex curvature tensor to the space of symmetric bivectors generated by vectors orthogonal to the space of nullity vectors at each point is a positive- or negative-definite Hermitian form on this space.*

Then M^n *is a metric product,* $M^n = C^\mu \times M^{n-\mu}$, *where* C^μ *is complete and flat, and* $M^{n-\mu}$ *is complete. Moreover,* C^μ *and* $M^{n-\mu}$ *are Kähler manifolds.*

COROLLARY. *The conclusion of Theorem* A *continues to hold if Condition* (A2) *is replaced by the condition that the curvatures of all holomorphic sections generated by vectors orthogonal to the space of nullity vectors at each point are* 1/2-*pinched.*

A Kähler immersion $\psi\colon M^d \to C^{d+k}$ is called n-*cylindrical* if $M^d = M^{d-n} \times C^n$ and $\psi = \overline{\psi} \times 1$, where 1 is the identity on C^n, ψ is a Kähler immersion of M^{d-n} into C^{d+k-n}, and C^m denotes complex m-space.

THEOREM B. *Let* M^d *be a complete, connected, and simply connected* C^∞ *Kähler manifold, Kähler-immersed in* C^{d+k} *with constant relative nullity* ν. *Then* ψ *is* ν-*cylindrical if one of the following conditions holds.*

(B1) $\nu = d - 2$.

(B2) *The curvatures of all holomorphic sections orthogonal to the nullity spaces are strictly negative.*

Nullity was defined by Chern and Kuiper [4]. Theorems A and B are Kähler analogues of similar theorems for Riemannian manifolds [5], [7]. I would like to thank Professor Yeaton H. Clifton for introducing me to the calculus of Kähler manifolds according to E. Cartan. Section (2) of this paper is a translation of this calculus into invariant language.

2. THE COMPLEX CURVATURE TENSOR

Let M be a Kähler manifold with almost-complex structure J [3]. We shall denote the tangent space to M at m by M_m, the Riemannian metric of M by $\langle\ ,\ \rangle$, the curvature transformation associated with the vectors x and y by R(x, y), the

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