THE CYCLIC CONNECTIVITY OF PLANE CONTINUA

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Suppose that p and q are distinct points of a locally connected, compact, plane continuum H. It is known that if no point separates p from q in H, then there exists a simple closed curve in H that contains both p and q [5]. It is also known that H is arcwise connected [4, Theorem 13, p. 91]. Recently, arcwise connectedness has been established for certain plane continua that are not locally connected [1]. A continuum M is said to be *aposyndetic at a point* p *of* M *with respect to a set* N *in* $M - \{p\}$ if there exist an open set U and a continuum H in M such that $p \in U \subset H \subset M - N$. A continuum M is said to be *aposyndetic at a point* p if for each point q in $M - \{p\}$, M is aposyndetic at p with respect to q. If M is aposyndetic at each of its points, then M is said to be *aposyndetic*. The arcwise connected continua studied in [1] may be classified as follows:

A compact plane continuum M is said to be of *type* 1 if it is aposyndetic and contains a finite set of points F with the property that for each point x in $M - F$, there exist points y and z in F such that M is not aposyndetic at x with respect to $\{y, z\}$.

If M is semilocally connected at all except finitely many of its points and if at none of its points it is both aposyndetic and semilocally connected, then M is said to be of *type* 2.

If M is a continuum of type 1, then it is the sum of a finite number of cyclicly connected continua [1, Theorem 6]. Hence any two distinct points of M that are not separated in M by a point lie inside a simple closed curve contained in M. However, if M is of type 2, then M may contain two points that are not separated in M by a point and are not contained in a simple closed curve lying in M [1, Example 8].

A compact plane continuum H that does not separate the plane has another cyclic property. A point r in $H - \{p, q\}$ is said to *cut* p *from* q *in* H if each subcontinuum of H that contains $\{p, q\}$ also contains r. F. Burton Jones has shown that if p and q are distinct points of H and no point cuts p from q in H, then some simple closed curve in H contains p and q [3]. It is known that if a compact plane continuum M contains a point y such that for each point x in $M - \{y\}$, M is semilocally connected at x and M is not aposyndetic at x with respect to y, then M has Jones's cyclic property [1, Theorem 12]. Note that M is a continuum of type 2. It is the primary purpose of this paper to show that all continua of type 2 have Jones's cyclic property. To accomplish this, we first establish a theorem from which it follows that all continua of type 1 or 2 are hereditarily arcwise connected. Also, we give an example that rules out certain generalizations of this result.

Throughout this paper, S denotes the set of points of a simple closed surface (that is, a 2-sphere). For definitions of unfamiliar terms and phrases, see [4].

Definition. Let F be a finite set of points $\{y_1, y_2, \cdots, y_\alpha\}$ in a continuum M $(M \subset S)$. For each i $(i = 1, 2, \cdots, \alpha)$, let $\{V_n^i\}$ be a properly nested sequence of

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