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THE ARITHMETIC OF THE TERM-RELATION NUMBER THEORY

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1. *Purpose*. In a previous paper [1], as a result of the formalization of the concept of internal relation, a term-relation number theory was introduced. The former work showed that term-relation numbers are either terms or relations obtained by following determined formation rules and by imposing certain postulates. Addition and multiplication of terms on the one hand, and relations on the other, were defined, and the following properties proved: associativity, commutativity, and distributivity of addition and multiplication; the existence of identity elements for addition $(0,\overline{0})$; the nonexistence of identity elements for multiplication; and the invalidity of the well-ordering principle for a concept of order similar to the one usually defined for natural numbers. The present paper will provide further definitions and examine further properties of term-relation numbers. These will include: definition of negative numbers; study of rings of term-relation numbers as partially ordered sets, leading to the characterization of such rings as modular lattices; definition of prime numbers; and study of divisibility and factorization. The paper will end with a question about the universal relevance of Weierstrass' final theorem of arithmetic.

2. *Terminology and notation*. The set of all terms T^∞, defined in [1], contains as a proper subset the set of all terms without proper components T^* ($T^* = \{0,1,2,\ldots\}$). The set of all relations R^∞, also defined in [1], contains as a proper subset the set of all relations without proper components R^* ($R^* = \{\overline{0},\overline{1},\overline{2},\ldots\}$). By "final component" of a term or relation, we mean a component without proper components, i.e., a component belonging either to T^* or to R^*. Every term or relation can be analyzed and expressed in its final components. Let us now introduce the set $T' = \{0,\pm 1,\pm 2,\ldots\}$, which can be obtained by an extension of T^* into a system of integers. Similarly, we obtain $R' = \{\overline{0},\pm\overline{1},\pm\overline{2},\ldots\}$ as an analogous extension of R^*. By "s.f.r," we mean the phrase "similarly for relations." Obviously, terms from T' (like terms from T^*) do not have proper components. S.f.r.

3. *Negative term-relation numbers*. If we let terms and relations have final components from T' and R', instead of from T^* and R^* only, we have

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