Notre Dame Journal of Formal Logic
Volume IV, Number 3, July 1963

A NOTE ON THE GENERALIZED CONTINUUM HYPOTHESIS. III.

BOLESŁAW SOBOCIŃSKI

§5*

In [11], p. 72, point (vii), and p. 76, point (xi), it is proved that the formulas *C1*, *B2* and *B3* which are the particular instances of formulas *C* and *B*, cf. [7], p. 274, are such that *C1* is a consequence of $\mathbf{E}_1$, *B2* follows from $\mathfrak{C}$ and *B3* is provable in the general set theory. Now we shall show:

1) that the following particular instances of *D*

D2 For any cardinal number $\mathfrak{m}$ *and any aleph* $\mathfrak{a}$, *if* $2^{\mathfrak{m}} = 2^{\mathfrak{a}}$, *then* $\mathfrak{m} = \mathfrak{a}$.

and

D3 For any cardinal number $\mathfrak{m}$ *and any aleph* $\mathfrak{a}$, *if* $2^{\mathfrak{m}} = 2^{2^{\mathfrak{a}}}$, *then* $\mathfrak{m} = 2^{\mathfrak{a}}$.

are consequences of Cantor's hypothesis on alephs.

2) that the following particular instance of *C*

C2 For any cardinal number $\mathfrak{m}$ *and any aleph* $\mathfrak{a}$, *if* $\mathfrak{a} < \mathfrak{m}$, *then* $2^{\mathfrak{a}} < 2^{\mathfrak{m}}$.

is a consequence of *D2*;

3) that the formulas *D1* and *C1*, which are, obviously, the instances of *D2* and *C2* respectively, are equivalent in the field of general set theory; and

4) that the following formula

$\mathbf{E}_3$ *For any cardinal number* $\mathfrak{m}$ *and any aleph* $\mathfrak{a}$, *if* $\mathfrak{m} < 2^{2^{\mathfrak{a}}}$, *then* $\mathfrak{m} \leqslant 2^{\mathfrak{a}}$

and which is such that $\mathbf{E}_2$ is its substitution follows from $\mathfrak{C}$.

We prove it as follows:

(xii) *Cantor's hypothesis on alephs implies formulas D2, D3 and* $\mathbf{E}_3$.

($\mathfrak{m}$) *Proof of D2.* Let us assume the conditions of *D2*, viz. that

*The first and the second parts of this paper appeared in *Notre Dame Journal of Formal Logic*, v. III (1962), pp. 274-278, and v. IV (1963), pp. 67-79. They will be referred to throughout this third part as [7] and [11] respectively. See the additional Bibliography given at the end of this part. An acquaintance with [7] and [11] is presupposed.

Received August 16, 1963