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SOME METHODS OF FORMAL PROOFS. III

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In [1] I gave a characterization of the first order functional calculus by means of a truncated definition of satisfiability; in [2] I gave a proof of completeness of the functional calculus of an arbitrary order (without the extensionality axiom and the definition axiom) in the non-standard sense. So there appears a question about an analogical characterization—by means of the truncated satisfiability definition—of the last calculus. This paper gives an answer to the question, that is, it gives a characterization of theses of the calculus of an arbitrary order by means of a truncated satisfiability definition.

We use the notation of [1] with the following:

$Q(k)$ - a set of tables of rank k,
$\{i_{w(F)}\}$ - indices of free variables occurring in F,
$w(F)$ - the number of free variables occurring in F,
$p(F)$ - the number of apparent variables occurring in F,
$n(F) = \max\{w(F) + p(F), \{i_{w(F)}\}\}$.

A non-standard model is defined by means of a sequence $\mathbf{M} = \langle B_1, \{F_q^t\}, B_2, \{G_q^t\}, B_3, \{H_q^t\}, \ldots\rangle$ where B_1 is a non-empty domain of arbitrary elements, B_2 is an arbitrary non-empty domain of relations on B_1, B_3 is a non-empty domain of relations of a given type and order which contains previously used relations of the type; and for each type there exists a domain B_i such that B_i is a non-empty domain of relations of the given type including previously used relations of the type; and $\{F_q^t\}$ is a sequence of relations from the domain B_2, $\{G_q^t\}$ is a sequence of relations from the domain B_3, and analogously for each sequence of relations of a given type. A model with finite domains is called a table; if domains of the table have k elements then it is called a table of rank k.

f - a function on domains of a table T with values in domains of a model $\mathbf{M}$.

A_k - a sample of k individuals and k relations of each type.

V_k - the set of all elements of each domain of a table in natural order.

$P(B, V_k)$ means: B is a permutation of V_k.

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