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COMBINATORIAL OPERATORS AND THEIR QUASI - INVERSES

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1. *Introduction*. Combinatorial operators were introduced by J. Myhill ([1], [2]) as a fundamental tool in the study of isols. A systematic exposition of those operators is given in the monograph [3] of J. Dekker, to which we refer for the notations. In [3], Dekker proved the following

Theorem 1.1. *Let ϕ be a combinatorial operator and ϕ^{-1} its quasi-inverse. If ϕ is recursive, then $\phi(\varepsilon)$ is a recursively enumerable set, and there is a partial recursive function χ, whose domain is $\phi(\varepsilon)$, such that*

$$\phi^{-1}(x) = \rho_{\chi(x)} \text{ for all } x \in \phi(\varepsilon). \tag{1.1.}$$

In this paper we investigate the measure in which the existence of a p.r. (partial recursive) function χ, such that $\phi^{-1}(x) = \rho_{\chi(x)}$ for all $x \in \phi(\varepsilon)$, determines the recursive character of the operator ϕ.

Besides the notations from [3], we shall use the following ones: $\langle \omega_i \rangle$, $i = 0,1, \ldots$, is the Post-enumeration of all r.e. (recursively enumerable) sets; F_R denotes the set of all r. (recursive) functions of one variable, and $\widetilde{F}_R$ denotes the set of all p.r. functions of one variable.

2. *The Fundamental Theorem*. Let ϕ be a combinatorial operator and ϕ_0 its dispersive operator. We shall say that ϕ (resp. ϕ_0) is *sub-effective* iff (if and only if) there is a disjoint r.e. sequence $\langle \omega_{\varphi_0(i)} \rangle_{i \in \varepsilon}$, $\phi_0 \in F_R$, of r.e. sets such that

$$\phi_0(\rho_n) \subset \omega_{\varphi_0(n)} \text{ for all } n \in \varepsilon. \tag{2.1.}$$

All theorems of this paper are, essentially, strengthenings of the following fundamental

Theorem 2.1. *A combinatorial operator ϕ is sub-effective iff there is a $\chi \in \widetilde{F}_R$ such that*

$$\phi^{-1}(x) = \rho_{\chi(x)} \text{ for all } x \in \phi(\varepsilon). \tag{2.2.}$$

Proof. Let ϕ be sub-effective and φ_0 as in (2.1). Then, $E = \bigcup_{i=0}^{\infty} \omega_{\varphi_0(i)}$ is a r.e. set and $\phi(\varepsilon) = \bigcup_{n=0}^{\infty} \phi_0(\rho_n) \subset E$ (where ϕ_0 is the dispersive operator of ϕ).

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