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NORMAL FORM GENERATION OF S5 FUNCTIONS VIA TRUTH FUNCTIONS

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1. *Generating the Singulary* S5 *Functions*. The fact that the number of n-ary S5 functions (connectives, if you prefer) is equal to the number of m-ary truth functions, where $m = 2^n + n - 1$ (Carnap [2] p. 48), has led Canty and Scharle in [1] to pose the fascinating problem now to be described. Consider the schema

$$T(p, F(p)).$$

Does there exist an S5 function F such that, as T runs through the binary truth functions, the expression $T(p, F(p))$ generates all the singulary S5 functions? Let us call such a function F a *solution to the singulary-function-generation problem*. In [1], Canty and Scharle correctly state that the modal function $\otimes_1$ is a solution to the singulary-function-generation problem (we use the names introduced in Massey [4] for the S5 functions), where the semantics of $\otimes_1$ is given by the following complete set of truth tables (Cf. Massey [4] on using complete sets of truth tables to define the semantics of an S5 connective):

p	$\otimes_1\ p$
t	t

p	$\otimes_1\ p$
f	f

p	$\otimes_1\ p$
t	f
f	t

It will be helpful in the sequel to stack these tables on top of one another thus:

p	$\otimes_1\ p$
t	t
f	f
t	f
f	t

Clearly a singulary S5 function F is a solution to the singulary-function-generation problem if and only if, for each pair (α, β) of truth values, there

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