Notre Dame Journal of Formal Logic
Volume XVI, Number 4, October 1975
NDJFAM

REMARK ON A LOGIC OF PREFERENCE

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Setsuo Saito [1] offers a logic of preference designed to circumvent some difficulties in Halldén's system $\mathcal{A}$. Saito's own system is not without problems, however. He himself notes that among the theses of his system are

$$pPt \equiv pP{\sim}p \tag{1}$$

and

$$tPq \equiv {\sim}qPq \tag{2}$$

where t is a tautology. Given the transitivity of preference, it follows from (1) and (2) that $pP{\sim}p$ and ${\sim}qPq$ entail pPq. But now let p stand for 'My arm is not cut off or my leg is cut off' and let q stand for 'I do not win a dime or I win ten pennies.' Presuming I am not in need of an amputation and that I would deem the loss of any one limb as tragic to me as a multiple loss of limbs, I may well prefer p over ${\sim}p$ on the grounds that the former could be true without my losing a limb while the latter could not. And I certainly prefer ${\sim}q$ over q since the former's, but not the latter's, being true assures me of being ten cents richer. But valuing my arms and legs as highly as I do, I most assuredly do not prefer p over q. At best I might be indifferent as between them.

Assuming that we do not want to give up the transitivity axiom, we must dispense with A9 and A10 from which (1) and (2) were derived. Since (1) and (2) do have some intuitive appeal when read as conditionals rather than equivalences, perhaps the least violence is done to Saito's system if we simply replace the two axioms by

$$\text{A9}' \qquad \Box(p \supset q) \supset (pPq \supset pP({\sim}p \cdot q))$$
$$\text{A10}' \qquad \Box(q \supset p) \supset (pPq \supset (p \cdot {\sim}q)Pq).$$

Whether the resulting system is of any particular interest is of course another question, one which must await an adequate semantical analysis of the preference relation.

Received September 21, 1973