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A GENERALISED PROPOSITIONAL CALCULUS

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1 *Introduction* Any propositional variable ambiguously denotes a statement, but it is also the case that any pair of *different* propositional variables are orthodoxly taken always to ambiguously denote statements *completely independently of each other*. In this paper we examine some of the consequences of suppressing the convention that they do so completely independently of each other. Of these, perhaps the most interesting is the undecidability of the two-valued propositional calculus which results when the propositional variables of the classical two-valued propositional calculus are replaced by a more general kind of variable which, although more general, still ranges over statements.

2 *Independence of variables* In this paper, the term "propositional variable" shall refer to any variable that ranges over statements and satisfies the following conditions. (1) To each pair of propositional variables there corresponds a (unique) mutual truth-table. This truth-table has 1, 2, 3 or 4 rows and 2 columns. The rows are all distinct. Each entry is a 1 or a 0, where 1 represents truth and 0 represents falsity. (2) Given any 2 propositional variables, consider the pairs of statements whose first member is one of the statements which the first propositional variable ambiguously denotes and whose second member is one of the statements which the second propositional variable ambiguously denotes at the same time. For each such pair of statements, the first statement must have a truth-value which occurs in the first column of the mutual truth-table for the 2 variables, and the second statement must have a truth-value which occurs in the second column and in a row containing, in its first column, the truth-value of the first statement.

The variables which are here referred to as "propositional variables" include all the propositional variables of the classical propositional calculus. In fact, the latter form a proper subset—that of those variables any pair of which always has a mutual truth-table made up of 4 rows. The following definition is therefore meaningful.

Definition Two propositional variables will be said to be *independent* if, and only if, their mutual truth-table has 4 rows.

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