Notre Dame Journal of Formal Logic
Volume XV, Number 4, October 1974
NDJFAM

A THEOREM CONCERNING A RESTRICTED RULE OF SUBSTITUTION IN THE FIELD OF PROPOSITIONAL CALCULI. II

BOLESŁAW SOBOCIŃSKI

6* It follows from definition Df.3, Remark IV and **5.4** that in $\mathfrak{D}_0$ for every m, $1 \leqslant m \leqslant y$, and for every k, $2 \leqslant k \leqslant z$, $\{\mathbf{s}_1^m\} \vdash_{\overline{\mathrm{R1}}} \mathbf{s}_k^m$, i.e., that $\mathbf{s}_k^m$ is a consequence by R1 of the first term of $\mathbf{S}_m$. We indicate by $\mathfrak{D}_1 = \{\mathbf{A};\ \mathbf{V}_{1\mathbf{E}}^*;\ \mathbf{V}_{2\mathbf{E}}^*;\ \mathbf{S}_1^+;\ \mathbf{S}_2;\ \ldots;\ \mathbf{S}_y\}$ an augmentation of $\mathfrak{D}_0$ such that $\mathfrak{D}_1$ is a proof sequence of $\mathbf{b}$ in which $\mathbf{S}_1^+ = \mathbf{S}_1$, but in which for every k, $1 < k < z$, there are two terms σ and τ such that they precede $\mathbf{s}_1^1$, i.e., the first term of $\mathbf{S}_1^+$, and $\{\sigma, \tau\} \vdash_{\overline{\mathrm{R2}}} \mathbf{s}_k^1$. In the other words, in $\mathfrak{D}_1$ every term of $\mathbf{S}_1$ is a consequence by R2 of two terms belonging to $\mathfrak{D}_1$ and preceding the first term of $\mathbf{S}_1$. Obviously, if $\mathbf{S}_1 = \{\mathbf{s}_1^1\}$, then $\mathfrak{D}_1 = \mathfrak{D}_0$. But, in such a case $\mathfrak{D}_0$ can be considered as a particular instance of $\mathfrak{D}_1$ which will not be analyzed separately. In a similar way we indicate by $\mathfrak{D}_2 = \{\mathbf{A};\ \mathbf{V}_{1\mathbf{E}}^*;\ \mathbf{V}_{2\mathbf{E}}^*;\ \mathbf{S}_1^{+*};\ \mathbf{S}_2^+;\ \ldots;\ \mathbf{S}_y\}$ an analogous augmentation of $\mathfrak{D}_1$ and so forth.

In this section we will prove that we can replace $\mathfrak{D}_0$ by its augmentation

$$\mathfrak{D}_y = \{\mathbf{A};\ \mathbf{V}_{1\mathbf{E}}^*;\ \mathbf{V}_{2\mathbf{E}}^*;\ \mathbf{S}_1^{+*};\ \ldots;\ \mathbf{S}_{y-1}^{+*};\ \mathbf{S}_y^+\}$$

such that $\mathfrak{D}_y$ is a proof sequence of $\mathbf{b}$ in which for every m, $1 \leqslant m \leqslant y - 1$, $\mathbf{A} \vdash_{\overline{\mathrm{R1^*,R2}}} \mathbf{S}_m^{+*}$ and, moreover, $\mathbf{A} \vdash_{\overline{\mathrm{R1^*,R2}}} \mathbf{S}_y^+$.

Since in order to prove this statement we shall use deductions entirely analogous to those that were presented in section **4**, the proof given below will be rather concise.

6.1 Let us assume that in $\mathfrak{D}_0$ $\mathbf{S}_1 \neq \{\mathbf{s}_1^1\}$ and, moreover, that $\mathbf{s}_k^1$, $2 \leqslant k \leqslant z$, is an arbitrary term of $\mathbf{S}_1$ such that $\mathbf{s}_1^1 \neq \mathbf{s}_k^1$. Then, *cf.*, **5.3** and Remark IV, in $\mathfrak{D}_0$ there are two terms σ and τ such that they precede $\mathbf{s}_1^1$, $\{\sigma, \tau\} \vdash_{\overline{\mathrm{R2}}} \mathbf{s}_1^1$ and $\{\mathbf{s}_1^1\} \vdash_{\overline{\mathrm{R1}}} \mathbf{s}_k^1$. Since, obviously, $\mathbf{s}_k^1$ is a substitution instance of $\mathbf{s}_1^1$, there is a

*The first part of this paper appeared in *Notre Dame Journal of Formal Logic*, vol. XV (1974), pp. 465-477. An acquaintance with that part and the reference given therein is presupposed.

Received January 23, 1972