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NECESSITY AND SOME NON-MODAL PROPOSITIONAL CALCULI

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Sometimes in a non-modal propositional calculus (**PC**) containing a connective (C) for implication a satisfactory definition of 'it is necessary that p'(Lp)' is available. Thus, in the well-known system E of entailment, Lp may be defined as $CCppp$, where 'C' denotes the non-truth-functional implication taken as a primitive connective. A non-modal **PC** may fail to permit an intuitively satisfactory definition of necessity either because it is too weak or because it is too strong. A non-trivial example of the former case is provided in [5], where the authors use the following four-valued model $\mathcal{N}$ (with starred elements as designated)

C	0	1	2	3
0	3	3	3	3
1	0	2	0	3
*2	0	3	2	3
*3	0	0	0	3

of the pure implicational calculus (**PIC**) P_I of ticket entailment defined in [1], to show that there is no pure implicational (**PI**) wff $\alpha(p)$ in the single variable p satisfying the following conditions:

(1) $C\alpha(p)p$ is a theorem of P_I,
(2) $Cp\alpha(p)$ is not a theorem of P_I,
(3) if β is a theorem of P_I, then $\alpha(p/\beta)$ is a theorem of P_I,

and

(4) for any δ, θ, $CC\delta\theta C\alpha(p/\delta)\alpha(p/\theta)$ is a theorem of P_I.

Corresponding to the modal axiom $CLCqrCLqLr$ consider now the condition

(4*) $C\alpha(p/C\delta\theta)C\alpha(p/\delta)\alpha(p/\theta)$ is a theorem of P_I.

Since transitivity of implication and modus ponens are available in P_I, if $\alpha(p)$ satisfies (4), in view of (1), it will also satisfy (4*). The authors of [5] are entitled to the following:

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