

## A NEW FAMILY OF MODAL SYSTEMS

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1 It is well-known that a Lemmon-style axiomatization of modal system S3 is provided in the following fashion:

### Formation Rules:

- (1) Any propositional variable is a wff.
- (2) If  $x$  is a wff, then so is  $Nx$ .
- (3) If  $x$  is a wff, then so is  $Lx$ .
- (4) If both  $x$  and  $y$  are wffs, then so is  $Cxy$ .
- (5) Nothing else is a wff.

### Axioms:

- A1  $CLpp$   
 A2  $CLCpqLCLpLq$

### Rules of Inference:

- (a) Uniform Substitution for Variables.
- (b) Detachment (D): If both  $Cxy$  and  $x$  are theses, then  $y$  is a thesis.
- (c) Restricted Necessitation (RN): If  $x$  is a PC-tautology or a thesis of the form  $CLyz$ , then  $Lx$  is a thesis. (This version of the rule of necessitation is taken from Zeman in [8], p. 105.)
- (d) Tautology Rule (PCR): If  $x$  is a PC-tautology, then  $x$  is a thesis.

### Definitions:

- B1  $Kxy =_{df} NCxNy$   
 B2  $Axy =_{df} NKNxNy$   
 B3  $Exy =_{df} KCxyCyx$   
 B4  $Mx =_{df} NLNx$   
 B5  $Fxy =_{df} LCxy$   
 B6  $Hxy =_{df} KFx y Fyx$

Now if we append to the above basis for S3 the additional axiom

- M1  $CNLpLNLp$

*Received March 3, 1975*