Notre Dame Journal of Formal Logic
Volume XVIII, Number 1, January 1977
NDJFAM

A STRONG COMPLETENESS THEOREM FOR 3-VALUED LOGIC: PART II

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Proof[1] was given in [1] that $\mathbf{SC}_3$, the 3-valued sentential calculus, has a strongly complete axiomatization. Pushing our investigation one step further,[2] we obtain here a like result about $\mathbf{QC}_3$, the 3-valued quantificational calculus of order one.[3]

1 The primitive signs of $\mathbf{QC}_3$ are

(a) '$\sim$', '$\supset$', '$\forall$', '(', ')', and ',',
(b) a denumerable infinity of individual variables, to be referred to by means of 'X',[4]
(c) a denumerable infinity of individual parameters, to be referred to by means of 'X',[5] and
(d) for each d from 0 on, a denumerable infinity of predicate parameters of degree d, to be referred to by means of 'F^d'.[6]

We presume the variables in (b), the parameters in (c), and the parameters in (d) to be alphabetically ordered; and we take the alphabetically first parameter of degree d in (d) to be 'p'.

The atomic wffs of $\mathbf{QC}_3$ are all formulas of the sort $\mathsf{F}^d(\mathsf{X}_1, \mathsf{X}_2, \ldots, \mathsf{X}_d)$, where F^d is a predicate parameter of degree d ($d \geqslant 0$) and $\mathsf{X}_1, \mathsf{X}_2, \ldots$, and X_d are individual parameters. The wffs of $\mathbf{QC}_3$ (presumed at one point below to be alphabetically ordered) are the atomic wffs just defined, plus all formulas of the sorts (i) $\sim A$, where A is well-formed, (ii) $(A \supset B)$, where A and B are well-formed, and (iii) $(\forall X)A$, where—for some individual parameter X—the result $A(\mathsf{X}/X)$ of replacing X everywhere in A by X is well-formed.[7] The length $\mathcal{L}(A)$ of an atomic wff is 1; the length $\mathcal{L}(\sim A)$ of a negation $\sim A$ is $\mathcal{L}(A) + 1$; the length $\mathcal{L}((A \supset B))$ of a conditional $(A \supset B)$ is $\mathcal{L}(A) + \mathcal{L}(B) + 1$; and the length $\mathcal{L}((\forall X)A)$ of a quantification $(\forall X)A$ is $\mathcal{L}(A(\mathsf{X}/X)) + 1$, where X is the alphabetically earliest individual parameter of $\mathbf{QC}_3$. We avail ourselves of the following ten abbreviations:

$$\begin{aligned} \text{`}\mathbf{f}\text{'} &=_{df} \text{`}\sim(\mathsf{p} \supset \mathsf{p})\text{'} \\ (A \vee B) &=_{df} ((A \supset B) \supset B)\text{[8]} \\ (A \,\&\, B) &=_{df} \sim(\sim A \vee \sim B) \\ (A \equiv B) &=_{df} ((A \supset B) \,\&\, (B \supset A)) \end{aligned}$$

Received May 24, 1974