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The Church-Rosser Theorem for the Typed λ-Calculus with Surjective Pairing

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Introduction The Church-Rosser theorem will be proved for a system obtained from the typed λ-calculus by adding pairing and projection constants. One of the rules for the pairing and projection constants implies that if t has a pair type, then there is a term formed by means of a pairing constant with which t is convertible. Consequently, following the usage of [1], the pairing considered is called 'surjective'.

Klop [2] showed that the Church-Rosser theorem fails for the type-free analog of the calculus considered here and for several simplified versions of that calculus. The result proved here together with the counterexamples of [2] furnish a demonstration that there is a difference in the behavior of typed and type-free combinatory systems with respect to the Church-Rosser property.

The methods of this paper are constructive. According to the referee, an unpublished paper of Girard's contains a proof of the result established here, but Girard's proof is not constructive.[1]

The proof given here is a simple extension of the one in [3]. In contrast with [3], detailed arguments by cases are omitted, and a notation (largely borrowed from [2]) which allows for diagrammatic presentation of arguments about reducibility is employed. Also, terms which are the same up to alphabetic change of bound variables are identified. Identity is expressed by '='. A couple of minor errors in [3] will be noted along the way.

1 ***The calculus*** *Terms* are built up from λ, parentheses, variables (denoted by 'x'), denumerably many pairing constants (denoted by 'P'), denumerably many left projection constants (denoted by 'L'), and denumerably many right projection constants (denoted by 'R'). Propositional formulas built up by means

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