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Semantic Paradox of Material Implication

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1 The classical paradoxes of material implication* are valid formulas of the truth-functional calculus which, like $p \rightarrow (q \rightarrow p)$ and $\sim p \rightarrow (p \rightarrow q)$ suggest that the material conditional is not an adequate rendering of the English "if . . . then . . .", nor perhaps of *any* sense of "implies".[1] The philosophical difficulty of assessing the significance of these formulas is aggravated by the formal fact that they all involve either the embedding of one conditional in another, or require the use of some further connective besides the conditional. The nesting of conditionals is a construction which is rare enough in natural languages that our intuitions about when such compounds are true are not reliable.[2] Where another connective is involved, it is clear that only the *joint* behavior of the conditional and the connective can be impugned. It is the purpose of this note to point out that the objectionable features of the truth-functional conditional are reflected in semantic features of the set of pure first-order conditionals (that is, sentences of the form $p \rightarrow q$ for primitive p and q), which involve no embedding or further connectives. In particular, any consistent assignment of truth values to those sentences determines the truth values of all of the primitive sentences. This is absurd, because no set of purely *hypothetical* facts should determine all of the *categorical* facts.

Consider a language with primitive propositional variables p, q, r, etc., and whose sole connective is the conditional $\rightarrow$. We suppose that it is partitioned into two sets Ct and Cf, the first consisting of all first-order conditionals which are taken as true, and the second consisting of the rest, which are taken to be false. The question is whether such a partition of the set C of all sentences of the form $p \rightarrow q$ determines truth values for all the propositional variables according to a given interpretation of the conditional. If it does, we will say that the partition $\langle Ct, Cf \rangle$ *spans* the language.

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