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First-Degree Entailments and Information

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Entailment is sometimes informally explained by saying that A entails B just in case B is contained in A. Pressed to explain this notion of containment, it seems plausible to begin by saying that B is contained in A just in case the information conveyed by B is included in that conveyed by A. This paper presents two interpretations of the first-degree (FD) entailments of propositional logic that are based directly on the notion of inclusion of information.* It is proved in Section 2 that one of these interpretations exactly characterizes the tautological entailments of [2], while the other exactly characterizes the valid arguments of classical truth-functional logic. In Section 3, following a line of reasoning suggested in part by consideration of these interpretations, it is argued that the claim that relevance logic better captures our intuitions about entailment than classical logic is false. Section 1 presents natural-deduction formulations of both classical and relevant FD entailments that are used in subsequent proofs.

1 Two systems of natural deduction rules I take $-$, $\vee$, and & as primitive connectives and assume that sentential letters are specified. Wffs are as usual. I let $A, B, \ldots, F$ (with or without numerical subscripts) range over wffs and let M and N range over finite nonempty sets of wffs.

Given any finite nonempty set of wffs M, an infinite set of wffs X_M is defined recursively as follows:

1. If $A \in M$, then $A \in X_M$.
2. $A \,\&\, B \in X_M$ iff $A \in X_M$ and $B \in X_M$.
3. If $A \in X_M$ or $B \in X_M$, then $A \vee B \in X_M$.

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