Notre Dame Journal of Formal Logic
Volume XXI, Number 2, April 1980
NDJFAM

SOME REMARKS ABOUT THE FAMILY $\mathcal{K}$ OF MODAL SYSTEMS

WLODZIMIERZ RABINOWICZ

1 *Introduction* $\mathcal{K}$-systems can be defined as follows:

D1. S is a $\mathcal{K}$-system $=_{df}$ S is deductively equivalent to some system S′ such that:

a. S′ is an extension of the propositional calculus (PC)
b. S′ is governed by the following rules of inference: the rule of uniform substitution, *modus ponens* (MP), and RL($\vdash\alpha \Rightarrow \vdash L\alpha$)
c. The following formulas are axioms of S′:

$$\left.\begin{array}{ll} \text{A1.} & Lp \supset p \\ \text{A2.} & L(p \supset q) \supset (Lp \supset Lq) \\ \text{A3.} & Lp \supset LLp \end{array}\right\} \text{ S4-axioms}$$

A4. $LMp \supset MLp$

d. Every other axiom of S′ is an S5-thesis
e. S5 is not contained in S′.

The following formulas have been used in the construction of $\mathcal{K}$-systems:

G1. $MLp \supset LMp$
D2. $L(Lp \supset Lq) \vee L(Lq \supset Lp)$
J1. $L(L(p \supset Lp) \supset p) \supset p$
H1. $p \supset L(Mp \supset p)$
F1. $L(Lp \supset q) \vee (MLq \supset p)$
R1. $p \supset (MLp \supset Lp)$.

As far as I know, only nine nonequivalent $\mathcal{K}$-systems have been so far described (see [2], [4], [7], [11], and [12]):

K1 = {S4; A4}
K2 = {S4; A4; G1}
K3 = {S4; A4; D2}
K1.1 = {S4; J1}
K2.1 = {S4; J1; G1}
K3.1 = {S4; J1; D2}
K1.2 = {S4; H1}

Received January 2, 1974; Revised December 10, 1979