Notre Dame Journal of Formal Logic
Volume XXI, Number 1, January 1980
NDJFAM

REFLECTIONS ON AN EXTENSIONALITY THEOREM

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1 Like any artifact, a formal language may not coincide with the intention with which it was constructed, either by having unanticipated but important properties or by lacking properties it was intended to have. Some theorems about a formal language assert, directly or indirectly, that the language has certain intended properties. Formulation of such a theorem brings it to explicit awareness that the language was to have a certain property. Recognition of the need for a proof of the theorem is the recognition that lacking a proof one simply does not know whether the language actually constructed is the language one intended to construct. The proof itself is a check that we have constructed what we intended to construct.

The theorem this paper investigates functions as a lemma in the soundness proof for the language $\mathcal{L}$ of Mates' rigorous and concise text *Elementary Logic* [1]. Its versions for other systems will be obvious enough not to require separate comment.

2 Of the logic texts with which I am familiar only Mates [1] explicitly notes this theorem. But it does not comment on its general content, significance, or need of proof even apart from its role in proving soundness. This paper aims at filling those gaps.

The proof of the theorem is both lengthy and complex (that is true of the only proof I have been able to devise; Mates has told me that his proof shares those properties) and is omitted here. My sole present concern is to reflect on the significance of the theorem and its need of proof.

I do not know whether these reflections belong only to the technical and not to the philosophical side of logic. My main points are ones of which I was long unaware and my conversations with other philosophers interested in logic lead me to guess that many of them may also be unware of these points. If this is so, there will be some value in making these points obvious.

3 $\mathcal{L}$ is a standard first-order language including predicates, individual constants and variables, connectives, and quantifiers. 'Formula', 'free

Received January 25, 1977