

ENTAILMENT AND PROOF

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1 *The proof, the thesis, and the paradox* Apparently, "Today is Monday and today is not Monday" does not entail "Plato loves Socrates". Yet the following proof that it does, looks valid. Hence the paradox.

Proof: Let " $p \cdot \neg p$ " stand for the first proposition, and " q " stand for the second. Then

- (1) $(p \cdot \neg p)$ entails p .
- (2) p entails $(p \vee q)$.
- (3) $(p \cdot \neg p)$ entails $\neg p$.
- (4) $\neg p$ and $(p \vee q)$ together entail q .
- (5) Applying the principle of transitivity of entailment to (1) and (2), we get: $(p \cdot \neg p)$ entails $(p \vee q)$.
- (6) Applying the principle of transitivity of entailment to (3), (5) and (4), we get: $(p \cdot \neg p)$ entails q .

(The principle of transitivity of entailment is: If P_1, \dots, P_n entail Q_1 , and P_1, \dots, P_n entail Q_2, \dots, Q_m , and if Q_1, \dots, Q_m entail R , then P_1, \dots, P_n entail R .)

It can be seen that the proof applies to all contradictions. Since it is adopted from Lewis [6], we shall call the thesis that any contradiction entails any proposition whatsoever, Lewis' thesis. We shall call the proof, Lewis' proof, and the paradox, Lewis' paradox.

The aim of the paper is to arrive at the *best* interpretation of entailment by analysing the role it plays in deductive proofs. We shall see that according to this interpretation, Lewis' proof is valid, the paradox is a misunderstanding, and that Lewis' thesis stands (quite contrary to our intuition).

2 *Reasons for the rejection of the thesis* Many philosophers reject Lewis' thesis. The major reasons are as follows:

- a. The thesis is counter-intuitive. It simply does not sound right to