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A MORE RELEVANT RELEVANCE LOGIC

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Relevant implication or entailment is designed to convey the notion of "logical consequence". While $A \supset B$, where $\supset$ is the weak implication usually specified by a truth table, tells us that either A is false or B is is true, $A \rightarrow B$, where $\rightarrow$ is entailment, tells us that A is actually used, and perhaps necessary, in the proof of B. In this paper we show that, in a certain sense, the entailment systems of Anderson and Belnap ([1]) are still not fully relevant and we describe a new system which is at least more so.

The simplest notion of relevance can be expressed in terms of the following deduction theorem:

If there is a proof of B using all of $A_1, \ldots, A_n$,
then $\vdash A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_n \rightarrow B$.

(The A_is, B, and all capital Roman letters used below range over (well formed) formulas as in [1]).

This deduction theorem together with modus ponens or $\rightarrow$E ($\rightarrow$elimination) is equivalent to the system $\mathbf{R}_{\rightarrow}$ of [1]. This system can be axiomatized as follows:

$R_{\rightarrow}1$ $\vdash A \rightarrow A$
$R_{\rightarrow}2$ $\vdash A \rightarrow B \rightarrow . B \rightarrow C \rightarrow . A \rightarrow C$
$R_{\rightarrow}3$ $\vdash (A \rightarrow . B \rightarrow C) \rightarrow . B \rightarrow . A \rightarrow C$
$R_{\rightarrow}4'$ $\vdash (A \rightarrow . B \rightarrow C) \rightarrow . A \rightarrow B \rightarrow . A \rightarrow C$[1]

The instance

$$\vdash A \rightarrow A \rightarrow . A \rightarrow A \tag{1}$$

of $R_{\rightarrow}1$ and $R_{\rightarrow}2$ lead directly to

$$\vdash A \rightarrow . (A \rightarrow A) \rightarrow A. \tag{2}$$

within which the $A \rightarrow A$ still seems to be irrelevant. Anderson and Belnap also claim (2) to be irrelevant, but perhaps for other reasons. They

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