

On the Relationship Between One-Point Frames and Degrees of Unsatisfiability of Modal Formulas

FABIO BELLISSIMA

Let L be a normal modal logic and χ_L the class of the frames on which it holds: χ_L determines, in the set $\mathcal{F}$ of all modal formulas, the subset Y of those formulas which are true in every frame of χ_L . From Y we can obtain the set $\neg Y$ of the negation of the formulas of Y , and then $\bar{Y}$ and $\neg\bar{Y}$; i.e., the complements of Y and $\neg Y$ in $\mathcal{F}$.

Up to this point the situation is like that of the Classical Propositional Calculus, where we have the sets T of the tautologies, $\neg T$ (formulas that are false under each valuation), $\bar{T}$ (false under at least one valuation), and $\neg\bar{T}$ (true under at least one valuation). Moreover, the truth-functionality of the classical connectives entails that these sets are the only sets of formulas that can be determined by taking into account the possible truth value of a formula with respect to the models of a given class, when we analyze the situation only by means of the words “for all”, “there exists”, “true”, and “false” referred to the models of the class. In fact we can consider all the models of the Classical Propositional Calculus to be built on a single frame with a single point: so the words “for all” and “there exists” can be referred only to the valuations.

In the case of a modal logic the situation is more involved; a formula ψ is true in a class χ_L of frames if: for each frame $\mathbf{A} \in \chi_L$, each valuation V on $\mathbf{A}$, and each point a of $\mathbf{A}$, $\langle \mathbf{A}, V \rangle \models \psi[a]$. By interchanging “for all” with “there exists”, or commuting the quantifier referring to the valuations with that referring to the points, or interchanging $\models$ with $\not\models$, we get many different sets of formulas determined by χ_L . These sets, which will be called U -sets (see Definition 2.1), indicate different degrees of unsatisfiability of a formula with respect to χ_L .

The first aim of this paper is to determine necessary and sufficient conditions which a class χ_L must satisfy in order that some U -sets coincide. Through

Received March 7, 1983