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A New Foundation for the Theory of Relations

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Relation algebras are characterized using certain multivalued algebraic systems called polygroupoids. The connection between these concepts provides a basis for an alternative to the usual approach to the study of relations. Examples of polygroupoids are given as well as an application to the theory of relations.

1 Introduction The purpose of this paper is to outline a new approach to the calculus of relations. Relation algebras were introduced by Tarski in [11] as an abstract algebraic system defined by a natural set of axioms. The principal models for these axioms are obtained from collections of binary relations on a set using the set-theoretic operations of union, intersection, relation composition, and converse. Such algebras are known as representable relation algebras. Not all models of Tarski's axioms are representable (cf. [9], [10]). The present study developed as an outgrowth of an investigation into ways of characterizing "nonrepresentable" relation algebras (cf. [3], [4]). The characterization given in Section 4 is an extension of the relationship (cf. [8], Section 5) between certain relation algebras and systems called Brandt groupoids. Nowadays, these systems are just called "groupoids" in category theory (cf. [6]). Basically, the idea in the treatment below is to replace the use of groupoids by multivalued groupoids in Tarski's complex algebra construction and thereby extend the relationship in [8] to all relation algebras.

The results in this paper can be developed using the language of category

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