

Verification, Falsification, and Cancellation in KT

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Abstract The main result of this paper is that KT (=T) is closed under a cancellation principle (if LA is provably equivalent to LB and MA is provably equivalent to MB then A is provably equivalent to B). This result extends to KTG1, but it does not extend to modal systems associated with the provability interpretation of L, such as KW (=G) and KT4Grz (=S4Grz). Following Williamson, these results are applied to philosophical concerns about the proper form for theories of meaning, via the interpretation of L as some kind of verifiability. The cancellation principle can then be read as saying that verifiability conditions and falsifiability conditions jointly determine truth conditions.

A modal logic S has the *single cancellation property* just in case $A \leftrightarrow B$ is a thesis of S whenever $LA \leftrightarrow LB$ is, where L is the necessity operator. S has the *double cancellation property* just in case $A \leftrightarrow B$ is a thesis of S whenever both $LA \leftrightarrow LB$ and $MA \leftrightarrow MB$ are, where M is the possibility operator. The main result of this note is that KT (=T), the smallest normal system to contain the T axiom $Lp \rightarrow p$, has the double (but not the single) cancellation property. Although these properties are mathematically quite natural, there is also a philosophical reason for investigating them, which it may be worthwhile to mention.

One can give a nonstandard interpretation of a modal logic S by reading L as an operator expressive of some kind of verifiability rather than of necessity. One could then say that formulas A and B have the same verifiability conditions according to S if and only if $LA \leftrightarrow LB$ is a thesis of S . By the redundancy property of truth, one can also say that A and B have the same truth conditions according to S if and only if $A \leftrightarrow B$ is a thesis of S . Thus S has the single cancellation property just in case formulas with the same verifiability conditions (ac-

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