

Truth Table Logic, with a Survey of Embeddability Results

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Abstract What logic is barely justified on the basis of the ‘meanings’ given to the connectives by the *left-right readings* of their truth tables?

The valid arguments involved in truth table computations are called *Kalmaric*. We set out a system T, consisting of normal proofs constructed by means of elegantly symmetrical introduction and elimination rules. In the system T there are two requirements, called (\Box) and ($>$), on applications of discharge rules. T is sound and complete for Kalmaric arguments. (\Box) requires nonvacuous discharge of assumptions; ($>$) requires that the assumption discharged be the sole one available of highest degree.

We then consider a ‘Duhemian’ extension T*, obtained simply by dropping the requirement ($>$). T* is a proper subsystem of intuitionistic relevant logic. Our main result is that T* is a double negation consistency companion to classical logic. Thus all one needs to add to T* to obtain classical logic is the (intuitionistic) absurdity rule, and the (classical) rule of double negation elimination. T* represents the inferential core that is justified by the left-right readings of the truth tables.

We survey all the embeddability results using various translation mappings “downwards” into subsystems of classical, intuitionistic, minimal, and intuitionistic relevant logic. This puts our main result into significant context.

1 How does one read off a logic from truth tables? It is often claimed that the standard two-valued truth tables for \sim , $\&$, \vee , and \supset capture the meanings

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