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BETH'S TABLEAUX FOR RELEVANT LOGIC

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1 *Sequents—Countermodels* For the concept of *relevant-model-structure* (r.m.s.) we* refer to [2]. Giving a r.m.s. $\mathfrak{M} = \langle K, 0, R, *\rangle$ satisfying postulates P_1-P_6 of [2], a "forcing relation" is defined between K and the set of atomic sentences defined in a given domain D such that:

$$\alpha \Vdash A \text{ and } R(0, \alpha, \beta) \Rightarrow \beta \Vdash A.$$

Then the forcing relation is extended according to usual stipulations:

$$\alpha \Vdash A \vee B \Leftrightarrow \alpha \Vdash A \text{ or } \alpha \Vdash B$$
$$\alpha \Vdash A \wedge B \Leftrightarrow \alpha \Vdash A \text{ and } \alpha \Vdash B$$
$$\alpha \Vdash A \rightarrow B \Leftrightarrow \forall \beta, \gamma, \beta \Vdash A \text{ and } R(\alpha, \beta, \gamma) \Rightarrow \gamma \Vdash B$$
$$\alpha \Vdash \overline{A} \Leftrightarrow \alpha^* \nVdash A$$
$$\alpha \Vdash \forall x F[x] \Leftrightarrow \forall a \in D, \alpha \Vdash F[a].$$

A *signed formula* is an expression of one of the forms:

$$+A \qquad -A$$

where A is a formula. A *sequent* is a finite sequence of signed sentences.

A *countermodel* of a sequent is given by:

A r.m.s. $\mathfrak{M} = \langle K, 0, R, *\rangle$.

A member α of K such that:

If $+A$ is in the sequent, then $\alpha \Vdash A$
If $-A$ is in the sequent, then $\alpha \nVdash A$.

$\langle \mathfrak{M}, \alpha\rangle$ is a *normal countermodel* if $\alpha = 0$ and $0^* = 0$. The sequent is said to be *valid* if it has no countermodel and *normally valid* if it has no normal countermodel.

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