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INTEGRAL TRANSFORMS OF FUNCTIONALS IN $L_2(C_0[0,T])$

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ABSTRACT. In this paper we give a necessary and sufficient condition that a functional $F(x)$ in $L_2(C_0[0,T])$ has an integral transform $\mathcal{F}_{\alpha,\beta}F(x)$ which also belongs to $L_2(C_0[0,T])$.

1. Introduction. Let $C_0[0,T]$ denote one-parameter Wiener space; that is, the space of all $\mathbf{R}$-valued continuous functions $x(t)$ on $[0,T]$ with $x(0)=0$. Let $\mathcal{M}$ denote the class of all Wiener measurable subsets of $C_0[0,T]$, and let m denote Wiener measure. $(C_0[0,T],\mathcal{M},m)$ is a complete measure space, and we denote the Wiener integral of a Wiener integrable functional F by

$$\int_{C_0[0,T]} F(x)m(dx).$$

Let $L_2(C_0[0,T])$ be the space of all real or complex-valued functionals F satisfying

$$\int_{C_0[0,T]} |F(x)|^2 m(dx) < \infty. \tag{1.1}$$

Let $K = K[0,T]$ be the space of $\mathbf{C}$-valued continuous functions defined on $[0,T]$ which vanish at $t=0$. Next we state the definition of the integral transform $\mathcal{F}_{\alpha,\beta}$ introduced in [**6**] and used in [**5**], for functionals F defined on K.

Definition 1. Let F be a functional defined on K. For each pair of nonzero complex numbers α and β, the integral transform $\mathcal{F}_{\alpha,\beta}F$ of F is defined by

$$\mathcal{F}_{\alpha,\beta}F(y) = \int_{C_0[0,T]} F(\alpha x+\beta y)m(dx), \quad y \in K, \tag{1.2}$$

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