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CONVERGENCE THEOREMS FOR ROWS OF HERMITE-PADÉ INTEGRAL APPROXIMANTS

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ABSTRACT. The object of this paper is the study of the convergence of integral approximants, which are a special case of Hermite-Padé approximants of Latin type, to functions which are analytic in a disk except for one interior singular point. We give detailed estimates of the rate of convergence of the sequence of approximants of type $[L/M;1]$ for fixed M, as $L \to \infty$, in a model case study. We also give estimates of the rate of convergence of approximants of type $[L/M;1;2]$ for fixed M, as $L \to \infty$, for a model exhibiting a confluent singularity. We prove that integral approximants of these types converge uniformly on compact subsets of the disk which is centered on the origin and has the singular point of the given function on its boundary. We further prove convergence on additional Riemann sheets beyond the principal one in a lune near the singular point.

1. Introduction. Functions which are defined on a multiply-connected Riemann surface can be approximated accurately only by functions having a similar Riemann surface. To this end, Hermite-Padé approximants (of Latin type) have been used successfully to approximate functions having branch cuts; these approximants were introduced by Padé [**23, 24**] and contemporaneously by Hermite [**14**]. Let $f(z)$ be a function which is analytic except for a finite number of branch points of square root type. Shafer studied the special case of quadratic approximants [**25**], which are suitable for $f(z)$. He showed how polynomials $P(z)$, $Q(z)$ and $R(z)$ can be found from a knowledge of $f(z)$ so that

$$P(z)y(z)^2 + Q(z)y(z) + R(z) = 0 \tag{1.1}$$

has a solution $y(z)$ which approximates $f(z)$ near its branch points. Provided that $P(z), Q(z)$ and $R(z)$ have been suitably chosen, the branch points of $y(z)$ will be located close to those of $f(z)$, and the Riemann surfaces of $f(z)$ and $y(z)$ will be similar.

Hermite-Padé approximation is the preferred method of approximation of a function $f(z)$ when $f(z)$ has known analytic properties (ideally, the topology of the Riemann surface of $f(z)$ should be known) and