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STABILITY OF INTERFACES WITH VELOCITY CORRECTION TERM

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ABSTRACT. We consider two-dimensional solidification problems in which both surface tension and dynamical undercooling are incorporated into the temperature condition at the interface. Our study indicates that the presence of a dynamical undercooling term does not alter the stability-instability spectrum. That is, a wave mode is unstable in the presence of the dynamical undercooling and surface tension terms if and only if it is unstable in the presence of the surface tension term alone. The *magnitude* of the exponential growth or decay, however, is strongly influenced by the presence of dynamical undercooling. Within the unstable mode regime, dynamical undercooling tends to decrease the magnitude of the instability. In this sense, then, it is a stabilizing influence. Within the stable mode region, the influence of the dynamical undercooling depends on the magnitude of the velocity.

1. Introduction. Mathematical models of solidification which include the Gibbs-Thomson equilibrium conditions have been studied for quite some time [**8,9**]. Here we study the shape stability of planar fronts in the context of a simplified nonequilibrium model which includes dynamical undercooling (interface attachment kinetics) [**1,4,6,11,12**, and for a more general model, **7**]. Specifically let the curve in $\mathbf{R}^2$, described by

$$x = R(y,t) \quad \text{or} \quad S(x,y,t) \equiv x - R(y,t) = 0 \tag{1.1}$$

be defined as the interface which separates the liquid and solid phases. With T denoting temperature (scaled so that it vanishes for a planar interface at equilibrium), $\hat{n} \equiv \overline{\nabla} s/|\nabla s|$ the unit normal (in the direction from solid to liquid), and ν the normal component of the velocity (positive if motion of the interface is toward the liquid side), one may

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