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STRONG SEMICONTINUITY FOR UNBOUNDED OPERATORS

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ABSTRACT. Let A be a C^*-algebra and A^{**} its enveloping von Neumann algebra. Pedersen and Akemann developed four concepts of lower semi-continuity for elements of A^{**}. Later, Brown suggested using only three classes: strongly lsc, middle lsc and weakly lsc. In this paper we generalize the concept of strong semi-continuity to the case of unbounded operators affiliated with A^{**}. First, we identify our generalized strongly lsc elements with weak* lsc affine $(-\infty, \infty]$-valued functions on $Q(A)$ vanishing at 0, and then we generalize various results of the theory of strong semicontinuity. Also, we discuss some interpolation problems and examples.

1. Introduction. In [**3**], Akemann and G. Pedersen defined four concepts of semi-continuity for elements of A^{**}, the enveloping von Neumann algebra of a C^*-algebra A. Later, L. Brown [**5**] suggested using only three classes $\overline{A^m_{sa}}$, $\tilde{A}^m_{sa}$, and $(\tilde{A}^m_{sa})^-$, and named them *strongly lsc, middle lsc* and *weakly lsc*, respectively. There are also three corresponding concepts of continuity, where "continuous" means "both lower and upper semi-continuous": the strong, respectively middle, weakly, continuous elements are elements in A_{sa}, respectively, $M(A)_{sa}$, $QM(A)_{sa}$. (All these terms are explained in Section 2.) Then L. Brown asked three questions, each of which is three-fold.

(Q1) Is every lsc element the limit of a monotone increasing net of continuous elements?

(Q2) Is every positive lsc element the limit of a monotone increasing net of positive continuous elements?

(Q3) If $h \geq k$, where h is lsc and k is usc, does there exist a continuous x such that $h \geq x \geq k$?

He provided reasonably satisfactory answers and made an extensive study on semi-continuity [**5**].

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